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Brief paper

Distributed formation tracking of networked mobile robots under unknown slippage effects☆

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ABSTRACT

A distributed formation tracking problem considering collision avoidance among robots is investigated for a class of networked mobile robots with unknown slippage effects. The position information of the leader robot is time-varying and accessible to only a small fraction of the follower robots. The skidding and slipping effects are considered in the kinematics and dynamics of multiple mobile robots and assumed to be unknown, together with the matrices of the robot dynamics. A distributed recursive design methodology using the function approximation technique is derived to design each follower robot's local controller with collision avoidance ability under directed networks. The main difficulty of this design is deriving the adaptive compensation laws of unknown slippage effects in order to achieve both formation tracking and collision avoidance by using one local controller for each follower under limited communication links. The boundedness of all signals in the networked closed-loop system and guaranteed collision avoidance among robots are established through Lyapunov stability analysis.

1. Introduction

Recursive and systematic design approaches such as backstepping (Krstic, Kanellakopoulos, & Kokotovic, 1995) and dynamic surface design (Swaroop, Hedrick, Yip, & Gerdes, 2000) have been widely used to design control systems of mobile robots at the dynamic level (see Do, Jiang, & Pan, 2008, Dong & Kuhnert, 2005, Fukao, Nakagawa, & Adachi, 2000, Jiang & Nijmeijer, 1997, and the references therein). These previous works generally disregarded slippage effects which should be considered in practical applications. Thus, significant research activities in modeling and control methodologies have been proposed in the presence of wheel skidding and slipping (Low & Wang, 2008; Wang & Low, 2008). To deal with unknown skidding and slipping, an adaptive tracking control approach was proposed for single mobile robots at the dynamic level (Yoo, 2010). In Yoo and Park (2013), a formation controller using a virtual structure (Do & Pan, 2007) was designed for multiple robots with slippage effects. However, this method has two restrictions: (i) each robot needs all robots' information to update its own state because of an inherent property of virtual structure approach; and (ii) the collision avoidance problem among robots is not considered.

Distributed consensus problems using the graph theory have received a great deal of attention from the control community due to the consideration of limited communication among agents. (See, for instance Hou, Cheng, & Tan, 2009, Liu, Chen, & Lu, 2009, Mei, Ren, & Ma, 2012, Olfati-Saber, Fax, & Murray, 2007, Ren & Beard, 2008, Su, Chen, Wang, & Lin, 2011, Yu, Chen, Cao, & Kurths, 2010, and the references therein). Meanwhile the collision avoidance problem among nonlinear agents with a lack of shared information has not been addressed. Besides, there are no other any research results on the distributed formation tracking and collision avoidance in the presence of slippage effects connected with nonlinearities unmatched in control inputs.

Motivated by these observations, this paper investigates a distributed formation tracking and collision avoidance problem of networked mobile robots with unknown slippage effects. The information of a time-varying leader robot is only available to a

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subset of follower robots. A distributed dynamic surface design methodology using the adaptive function approximation technique is derived to design the potential-function-based local controller using only neighbors' information under directed networks. The contributions of this paper are three-fold: (i) compared with the existing literature, the distributed collision avoidance problem in the presence of limited communication links is considered for networked nonlinear mobile robots for the first time; (ii) adaptive laws for compensating unknown slippage effects are derived to achieve both distributed formation tracking and collision avoidance by using one local controller to each follower robot in directed networks; and (iii) information on all matrices of the robot dynamics and all slippage effects is not required to implement the proposed control system. The stability of the controlled closed-loop system is analyzed through Lyapunov stability theorem.

2. Problem formulation

2.1. Graph theory notions

Let $\mathcal{G} \triangleq (\mathcal{V}, \mathcal{E})$ be a directed graph with the set of nodes or vertices $\mathcal{V} \triangleq \{1, \dots, M\}$ and the set of edges or arcs $\mathcal{E} \subseteq \mathcal{V} \times \mathcal{V}$. An edge $(j, i) \in \mathcal{E}$ means that agent i can obtain information from agent j, but not vice versa where j and i are the parent node and child node, respectively. The set of neighbors of a node i is $\mathcal{N}_i = \{j | (j, i) \in \mathcal{E}\}$, which is the set of nodes with edges incoming to node i. A directed path from node i_1 to node i_k is a sequence of edges of the form $(i_1, i_2), (i_2, i_3), \dots, (i_{k-1}, i_k)$ in a directed graph. A directed tree is a directed graph where every node has one parent except for the root and the root has directed paths to every other node. A directed graph has a directed spanning tree if there exists at least one agent that has directed paths to all other agents.

2.2. Kinematics and dynamics of networked mobile robots in the presence of slippage effects

Suppose that there exist M followers, labeled as agents 1 to M, and a leader, labeled as an agent 0 under a directed graph topology. A group of the followers consists of M wheeled mobile robots in the presence of the wheel's skidding and slipping described by Yoo (2010)

$$\dot{q}_i = J_i(q_i)(z_i - \xi_i) + \varphi_i(q_i, \mu_i), \tag{1}$$

$$\tau_i = H_i(\dot{z}_i - \dot{\xi}_i) + C_{i,1}(\dot{q}_i)(z_i - \xi_i) + C_{i,2}(q_i)\dot{\varphi}_i(q_i, \mu_i) + C_{i,3}(\dot{q}_i)\varphi_i(q_i, \mu_i) \tag{2}$$

where $i = 1, 2, \dots, M$, $q_i = [x_i, y_i, \theta_i]^\top \in \mathbb{R}^3$; (x_i, y_i) are the coordinates of the midpoint between two driving wheels of the ith robot, θ_i is the heading angle of the ith robot, $z_i = [v_i, \omega_i]^\top \in \mathbb{R}^2$; v_i and ω_i are a forward linear and an angular velocity of the ith robot, respectively, $\xi_i = [\xi_{i,1}, \xi_{i,2}]^\top \in \mathbb{R}^2$; $\xi_{i,1}$ is the longitudinal slip velocity of the ith robot and $\xi_{i,2}$ is the yaw rate perturbation due to the wheel slippage of the ith robot, $\varphi_i = [-\mu_i \sin\theta_i, \mu_i \cos\theta_i, 0]^\top \in \mathbb{R}^3$; μ_i denotes the lateral skidding velocity of the ith robot, $\tau_i = [\tau_{i,1}, \tau_{i,2}]^\top \in \mathbb{R}^2$ is the control torque applied to the wheels of the ith robot, and $J_i(q_i) = \begin{bmatrix} \cos\theta_i & \sin\theta_i & 0 \\ 0 & 0 & 1 \end{bmatrix}^\top$. The definitions of the matrices H_i, $C_{i,1}(\dot{q}_i)$, $C_{i,2}(q_i)$, and $C_{i,3}(\dot{q}_i)$ in the dynamics (2) are given in Yoo and Park (2013). Each follower described by (1) and (2) can consist of heterogeneous kinematics and dynamics.

Assumption 1 (*Wang & Low, 2008*)**.** The skidding and slipping perturbations $\xi_{i,1}$, $\xi_{i,2}$, and $\varphi_i(q_i, \mu_i)$ are bounded as $|\xi_{i,1}| < \beta_{i,1}$, $|\xi_{i,2}| \le \beta_{i,2}$, and $\|\varphi_i(q_i, \mu_i)\| = |\mu_i| < \beta_{i,3}$ due to $\|[-\sin\theta_i, \cos\theta_i]^\top\| = 1$, respectively, where $\beta_{i,j} > 0$, $j = 1, 2, 3$, are *unknown* constants. Their first derivatives are bounded as $|\dot{\xi}_{i,1}| < \beta_{i,d1}$, $|\dot{\xi}_{i,2}| < \beta_{i,d2}$, and $\|\dot{\varphi}_i(q_i, \mu_i)\| < \beta_{i,d3} + \beta_{i,3}(|\omega_i| + \beta_{i,2})$ with $|\dot{\mu}_i| < \beta_{i,d3}$ where $\beta_{i,dj} > 0$, $j = 1, 2, 3$, are *unknown* constants.

Assumption 2. The system matrices H_i, $C_{i,1}(\dot{q}_i)$, $C_{i,2}(q_i)$, and $C_{i,3}(\dot{q}_i)$ for the dynamics (2) are *unknown* where $i = 1, \dots, M$.

The communication topology for the $M + 1$ robots is described by a directed graph $\mathcal{G} \triangleq (\mathcal{V}, \mathcal{E})$ with $\mathcal{V} \triangleq \{0, 1, 2, \dots, M\}$. To represent the communication among follower robots, we define a subgraph as $\bar{\mathcal{G}} \triangleq (\bar{\mathcal{V}}, \bar{\mathcal{E}})$ with $\bar{\mathcal{V}} \triangleq \{1, 2, \dots, M\}$. The adjacency matrix $\bar{\mathcal{A}}$ of the subgraph $\bar{\mathcal{G}}$ is $\bar{\mathcal{A}} = [a_{ij}] \in \mathbb{R}^{M\times M}$; $a_{ij} > 0$ if $(j, i) \in \bar{\mathcal{E}}$, $a_{ij} = 0$ otherwise, and $a_{ii} = 0$. Then, the (nonsymmetric) Laplacian matrix $\mathcal{L}$ of $\mathcal{G}$ is defined as $\mathcal{L} = \begin{bmatrix} 0 & 0_{1\times M} \\ -b & \bar{\mathcal{L}} + \mathcal{B} \end{bmatrix}$ where $b = [b_1, \dots, b_M]^\top$, with $b_i > 0$ if the leader robot $0 \in \mathcal{N}_i$ and $b_i = 0$ otherwise, denotes the communication weight from the leader robot to the follower robots, $\mathcal{B} = \mathrm{diag}[b_1, \dots, b_M]$, and $\bar{\mathcal{L}} = \bar{\mathcal{D}} - \bar{\mathcal{A}}$ is the Laplacian matrix of the subgraph denoting the communication among followers. Here, $\bar{\mathcal{D}} = \mathrm{diag}[d_1, \dots, d_M]$; $d_i = \sum_{j=1, j\neq i}^{M} a_{ij}$ is the diagonal element of the degree matrix $\bar{\mathcal{D}}$.

Remark 1. If the directed graph $\mathcal{G}$ has a spanning tree, $\mathrm{rank}(\mathcal{L}) = M$ (Ren, 2008). Then, $\mathrm{rank}(\bar{\mathcal{L}} + \mathcal{B}) = M$ from $(\bar{\mathcal{L}} + \mathcal{B})1_M = b$ where 1_M is an M-vector of all ones.

2.3. Potential function for collision avoidance

The potential function for collision avoidance among robots is defined as

$$P_{i,h} = \left(\min\left\{0, \frac{\varsigma_{i,h}^2 - \bar{L}_{i,h}^2}{\varsigma_{i,h}^2 - l_{i,h}^2}\right\}\right)^2 \tag{3}$$

where $i = 1, \dots, M$, $h = 1, \dots, M$, $h \neq i$, $\varsigma_{i,h} = \sqrt{(x_i - x_h)^2 + (y_i - y_h)^2}$ denotes the distance between the ith robot and the hth robot, $\bar{L}_{i,h} = \max\{L_i, L_h\} > 0$; L_i and L_h are the radii of the detection region of the ith robot and the hth robot, respectively, and $l_{i,h} = l_{h,i} > 0$ is the avoidance region denoting the smallest safe distance between the robots and $\bar{L}_{i,h} > l_{i,h} > 0$.

The partial derivatives of $P_{i,h}$ with respect to the x_i and y_i coordinates are defined as

$$\frac{\partial P_{i,h}}{\partial x_i} = \begin{cases} \bar{P}_{i,h}(x_i - x_h), & \text{if } l_{i,h} < \varsigma_{i,h} < \bar{L}_{i,h}; \\ 0, & \text{otherwise,} \end{cases} \tag{4}$$

$$\frac{\partial P_{i,h}}{\partial y_i} = \begin{cases} \bar{P}_{i,h}(y_i - y_h), & \text{if } l_{i,h} < \varsigma_{i,h} < \bar{L}_{i,h}; \\ 0, & \text{otherwise,} \end{cases} \tag{5}$$

where $\bar{P}_{i,h} = 4\frac{(\bar{L}_{i,h}^2 - l_{i,h}^2)(\varsigma_{i,h}^2 - \bar{L}_{i,h}^2)}{(\varsigma_{i,h}^2 - l_{i,h}^2)^3}$.

2.4. Problem statement

The control objective is to design the approximation-based distributed adaptive formation control laws τ_i for networked mobile robots (1) and (2) under a direct graph so that

(I) outside the detection range ($\varsigma_{i,h} \ge \bar{L}_{i,h}$), $\lim_{t\to\infty} \|\psi_i(t) - l(t) - \Delta_{i0}\| < \bar{\epsilon}$ and $\lim_{t\to\infty}(\sin(\theta_i(t) - \theta_0(t)) + \frac{\mu_i}{v_0}) \le \iota_i$ where $\psi_i = [x_i, y_i]^\top$, $\bar{\epsilon} > 0$ and $\iota_i > 0$ are constants that can be made sufficiently small, $\Delta_{i0} = \Delta_i - \Delta_0 \in \mathbb{R}^2$ denotes the desired offset between the ith follower robot and the leader robot, and $l(t) = [x_0(t), y_0(t)]^\top \in \mathbb{R}^2$ is the leader's position generated by $\dot{x}_0 = v_0 \cos(\theta_0)$, $\dot{y}_0 = v_0 \sin(\theta_0)$, and $\dot{\theta}_0 = \omega_0$ with the position (x_0, y_0) and orientation θ_0 and the linear and angular velocities ($v_0 > 0$, ω_0); and