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Brief paper

Passification-based adaptive control: Uncertain input and output delays[☆]

Anton Selivanov[a], Emilia Fridman[a], Alexander Fradkov[b,c]

[a] School of Electrical Engineering, Tel Aviv University, Tel Aviv, Israel

[b] Department of Theoretical Cybernetics, St. Petersburg State University, St. Petersburg, Russia

[c] Institute for Problems of Mechanical Engineering, Russian Academy of Sciences, St. Petersburg, Russia

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ABSTRACT

For a class of uncertain systems we analyze passification-based adaptive controller in the presence of small, unavoidable input and output time-varying delays as may be present in controller implementation. We derive upper bounds for time delays such that in some domain of initial conditions the states of the closed-loop system tend to zero, whereas an adaptive controller gain tends to a constant value. The results are semi-global, that is the domain of initial conditions is bounded but can be made arbitrary large by tuning an appropriate controller parameter. For the first time, we apply an adaptive controller to linear uncertain networked control systems, where sensors, controllers, and actuators exchange their information through communication networks. The efficiency of the results is demonstrated by the example of adaptive network-based control of an aircraft.

1. Introduction

In this paper we consider passification-based adaptive controller, which proved to be efficient for stabilization of delay-free systems. As it has been shown in Fradkov (1974), any hyper-minimum-phase linear time-invariant system can be stabilized by a static output feedback $u(t) = -ky(t)$ if k is large enough (for more established description see Andrievskii and Fradkov (2006)). For the case of uncertain systems an adaptive version of this controller has been derived via the speed gradient method (Fradkov, 1980).

While applying adaptive controller it is important to take into account unknown unavoidable input/output delays, which is a challenging problem (Krstic, 2010; Tsykunov, 1984). Most of the existing works on adaptive control deal only with state delays, e.g. (Ben Yamin, Yaesh, & Shaked, 2010; Mirkin & Gutman, 2004, 2010; Zhang, Xu, & Chu, 2010) to name a few. Adaptive controllers for linear systems with full state measurements and a *constant* input delay have been proposed and analyzed in Dydek, Annaswamy, Slotine, and Lavretsky (2013); Toodeshki, Hosseinnia, and Askari (2011). Passification-based adaptive *output*-feedback controller with a constant input delay has been studied in Mizumoto (2013).

Note that for linear time-invariant systems with constant time-delays there is almost no difference between an input and output delay, since the transfer function is the same. A more challenging problem is adaptive stabilization with *time-varying* delays, where input and output delays should be treated separately. A possible way to approach this problem is to assume that the difference between current and delayed signal is small enough (Balas & Nelson, 2011; Nelson, Balas, & Erwin, 2013), but this assumption is restrictive and difficult to verify.

In the present paper we suggest a simple adaptive output-feedback controller that stabilizes hyper-minimum-phase systems with input and output time-varying delays. Namely, we derive upper bounds on the time-delays such that in a given domain of initial conditions the states of the closed-loop system tend to zero, whereas an adaptive controller gain tends to a constant value. By changing a particular controller parameter the domain of acceptable initial conditions can be made arbitrary large leading to semi-global stability (see Remark 2). Moreover, we consider fast-varying delays (without any constraints on the delay-derivatives). This al-

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E-mail addresses: antonselivanov@gmail.com (A. Selivanov), emilia@eng.tau.ac.il (E. Fridman), fradkov@mail.ru (A. Fradkov).

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lows to apply, for the first time, an adaptive controller to linear uncertain networked control systems, where variable sampling intervals and communication delays are taken into account (see Section 4). Some preliminary results (without input delays) have been presented in Selivanov, Fridman, and Fradkov (2013).

Notations: Throughout the paper the superscript "T" stands for matrix transposition, $\mathbb{R}^n$ denotes the n dimensional Euclidean space with vector norm $\|\cdot\|$, $\mathbb{R}^{n\times m}$ is the set of all $n\times m$ real matrices, the notation $P>0$ for $P\in\mathbb{R}^{n\times n}$ means that P is symmetric and positive definite, $\lambda_{\min}(P)$ and $\lambda_{\max}(P)$ stand for the minimum and maximum eigenvalues of the matrix P, respectively. The symmetric elements of the symmetric matrix will be denoted by $*$. The set $\{0,1,2,\ldots\}$ is denoted by $\mathbb{Z}_+$.

2. Preliminaries and problem formulation

2.1. Preliminaries: Passification method

For non-delay linear time-invariant systems passification method and the corresponding design of an adaptive controller are based on Passification lemma that we state below.

Definition 1. For given matrices $A\in\mathbb{R}^{n\times n}$, $B\in\mathbb{R}^{n\times 1}$, $C\in\mathbb{R}^{l\times n}$, $g\in\mathbb{R}^{l\times 1}$ a transfer function $g^TW(s)=g^TC(sI-A)^{-1}B$ is called *hyper-minimum-phase (HMP)* if the polynomial $\varphi(s)=g^TW(s)\det(sI-A)$ is Hurwitz and g^TCB is a positive number.

Lemma 1 (*Passification Lemma, Fradkov, 1976*)**.** *Let the matrices $A\in\mathbb{R}^{n\times n}$, $B\in\mathbb{R}^{n\times 1}$, $C\in\mathbb{R}^{l\times n}$, $g\in\mathbb{R}^{l\times 1}$ be given. Then for existence of $P\in\mathbb{R}^{n\times n}$ and $k_*\in\mathbb{R}$ such that*

$$P>0,\quad PA_*+A_*^TP<0,\qquad PB=C^Tg, \tag{1}$$

where $A_*=A-Bk_*g^TC$, it is necessary and sufficient that the function $g^TW(s)=g^TC(sI-A)^{-1}B$ is HMP.

An appropriate value for k_* in Lemma 1 is any positive number such that

$$k_*>-\inf_{\omega\in\mathbb{R}}\operatorname{Re}\left\{\left(g^TW(i\omega)\right)^{-1}\right\}. \tag{2}$$

See Andrievskii and Fradkov (2006) for more details on Passification method.

2.2. Problem formulation

Consider an uncertain linear system

$$\begin{aligned}&\dot x(t)=A_\xi x(t)+Bu(t-r_1(t)),\quad x(0)=x_0,\\ &y(t)=Cx(t-r_2(t)),\end{aligned} \tag{3}$$

where $x\in\mathbb{R}^n$ is the state, $u\in\mathbb{R}$ is the control input, $y\in\mathbb{R}^l$ is the measurable output; A_ξ is an uncertain matrix that resides in the polytope

$$A_\xi=\sum_{i=1}^N\xi_iA_i,\quad 0\le\xi_i\le 1,\quad \sum_{i=1}^N\xi_i=1. \tag{4}$$

The delays $r_1(t), r_2(t)$ are supposed to be unknown and bounded:

$$0\le r_1(t)\le h_1,\qquad 0\le r_2(t)\le h_2.$$

We set $x(t)=0$ for $t<0$. This does not affect the solution $x(t)$ and implies that $y(t)=0$ if $t-r_2(t)<0$.

Denote

$$r(t)=r_1(t)+r_2(t-r_1(t)). \tag{5}$$

The quantity $r(t)$ is the overall delay of the closed-loop system. Clearly

$$r(t)\le h_1+h_2\triangleq h.$$

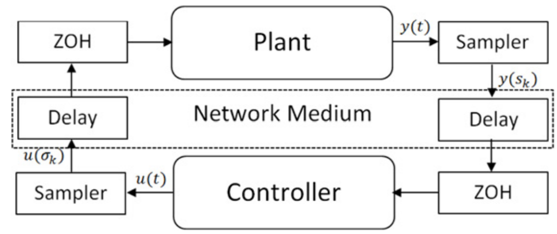

Fig. 1. Networked control system.

If $t-r(t)<0$ then the system (3) is in the open-loop since it has not received a signal from the controller. Therefore, a special analysis is needed on the intervals where $t-r(t)<0$. Following (Liu & Fridman, 2014) we assume

Assumption 1. There exists a unique $t_*>0$ such that

$$\begin{cases}t-r(t)<0, & t<t_*,\\ t-r(t)\ge 0, & t\ge t_*.\end{cases}$$

Assumption 1 has a simple physical meaning: the system (3) starts to receive signals from the controller at time t_*. It is clear that $t_*\le h$. Assumption 1 is always satisfied for slowly-varying delays with $\dot r(t)\le 1$ (since $t-r(t)$ is increasing) and for networked control systems as considered in Section 4.

Similar to Andrievskii and Fradkov (2006); Fradkov (1976) we assume

Assumption 2. There exists a known $g\in\mathbb{R}^l$ such that $g^TC(sI-A_\xi)^{-1}B$ is HMP for all A_ξ from (4).

For a given g satisfying Assumption 2 we consider the adaptive controller

$$\begin{aligned}&u(t)=-k(t)g^Ty(t),\\ &\dot k(t)=\gamma^{-2}\left(g^Ty(t)\right)^2,\end{aligned} \tag{6}$$

where $k,\gamma\in\mathbb{R}$, $\gamma>0$.

For $r_1(t)=r_2(t)\equiv 0$ under Assumption 2 it has been shown in Andrievskii and Fradkov (2006) that solutions of the closed-loop system (3), (6) satisfy the following property: for all $k(0)\in\mathbb{R}$

$$\lim_{t\to\infty}\|x(t)\|=0,\qquad \lim_{t\to\infty}k(t)=\text{const}. \tag{7}$$

Our objective is to derive conditions ensuring (7) for non-zero delays and for a certain choice of $k(0)$.

3. Main results

The closed-loop system (3), (6) can be presented in the form

$$\begin{aligned}\dot x(t)&=A_\xi x(t)-k_*Bg^TCx(t-r(t))\\ &\quad+(k_*-k(t-r_1(t)))Bg^TCx(t-r(t)),\\ \dot k(t)&=\gamma^{-2}\left(g^TCx(t-r_2(t))\right)^2\end{aligned} \tag{8}$$

with $k(t)=k(0)$ for $t<0$. Note that here $\dot x(0)$ and $\dot x(t_*)$ denote right-hand side derivatives.

The idea of passification-based approach is the following. Under Assumption 2 there exist $P>0$, k_* that satisfy (1). Consider a Lyapunov-like function

$$V_0(x,k)=x^TPx+\gamma^2(k-k_*)^2.$$

Its derivative along the trajectories of (8) has the form

$$\begin{aligned}\dot V_0&=2x^T(t)P\left[A_\xi x(t)-k_*Bg^TCx(t-r(t))\right]\\ &\quad+2(k_*-k(t-r_1(t)))x^T(t)PBg^TCx(t-r(t))\\ &\quad+2(k(t)-k_*)\left(g^TCx(t-r_2(t))\right)^2.\end{aligned} \tag{9}$$