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## Brief paper Decentralized robust adaptive output feedback control of stochastic nonlinear interconnected systems with dynamic interactions<sup> $\hat{z}$ </sup>

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#### a r t i c l e i n f o

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#### A B S T R A C T

For a class of stochastic nonlinear large-scale systems with output dynamic interactions, the problem of robust adaptive control is researched in this paper. Under the assumptions that the inverse dynamics of the subsystems are stochastic input-to-state stable, a decentralized robust adaptive output feedback controller is designed by combining the backstepping method and changing supply functions technique. Under some milder conditions, it is shown that the outputs can be regulated into an arbitrarily small neighborhood of the origin in probability and the other signals in the closed-loop system are globally bounded in probability by tuning design parameters. Moreover, when the drift and diffusion terms vanish at the origin, the equilibrium of the closed-loop system is globally stable in probability and the outputs tend to the origin almost surely. A simulation example is presented to demonstrate the effectiveness of the designed controller.

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#### **1. Introduction**

Large-scale systems can be found frequently in practical applications such as power supply systems, computer and telecommunication networks, economic systems and multiagent systems. For the interconnected systems, decentralized control is an efficient and practical technique where the design of total controller is simplified into ones of subsystems. However, simplicity of the design makes the stability analysis much more difficult. Research on decentralized adaptive control for large-scale systems by using backstepping technique has received great attention. In [Wen](#page--1-2) [\(1994\)](#page--1-2), the first result on decentralized adaptive control using such a technique was reported. In [Wen,](#page--1-3) [Zhou,](#page--1-3) [and](#page--1-3) [Wang](#page--1-3) [\(2009\)](#page--1-3), based on the backstepping approach, the decentralized adaptive controllers are designed for both linear and nonlinear systems with interactions directly depending on subsystem inputs and outputs. For other developments in this direction, please refer to [Guo,](#page--1-4) [Jiang,](#page--1-4) [and](#page--1-4) [Hill](#page--1-4) [\(1999\)](#page--1-4), [Jain](#page--1-5) [and](#page--1-5) [Khorrami\(1997\)](#page--1-5), [Jiang](#page--1-6) [and](#page--1-6) [Repperger](#page--1-6) [\(2001\)](#page--1-6), [Jiang](#page--1-7)

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<http://dx.doi.org/10.1016/j.automatica.2015.01.017> 0005-1098/© 2015 Elsevier Ltd. All rights reserved. [\(2000\)](#page--1-7), [Liu](#page--1-8) [and](#page--1-8) [Li](#page--1-8) [\(2002\)](#page--1-8), [Liu](#page--1-9) [and](#page--1-9) [Xie](#page--1-9) [\(2010\)](#page--1-9), [Wen](#page--1-10) [\(1995\)](#page--1-10), [Wen](#page--1-11) [and](#page--1-11) [Soh](#page--1-11) [\(1997;](#page--1-11) [1999\)](#page--1-12), [Wen](#page--1-13) [and](#page--1-13) [Zhou](#page--1-13) [\(2007\)](#page--1-13), [Zhang,](#page--1-14) [Wen,](#page--1-14) [and](#page--1-14) [Soh](#page--1-14) [\(2000\)](#page--1-14), [Zhou](#page--1-15) [and](#page--1-15) [Wen](#page--1-15) [\(2008\)](#page--1-15), and the references therein. All the results focus on deterministic large-scale systems.

In [Xie](#page--1-16) [and](#page--1-16) [Xie](#page--1-16) [\(2000\)](#page--1-16), a global decentralized stabilization scheme was proposed for a class of large-scale stochastic nonlinear systems where the drift and diffusion vector fields depend only on measurable outputs. In [Arslan](#page--1-17) [and](#page--1-17) [Basar](#page--1-17) [\(2003\)](#page--1-17), a class of strict-feedback systems which interact through their outputs was considered with the known diffusion. Recently, [Liu,](#page--1-18) [Zhang,](#page--1-18) [and](#page--1-18) [Jiang](#page--1-18) [\(2007\)](#page--1-18) investigated the problem of decentralized adaptive output-feedback stabilization for a class of large-scale stochastic nonlinear systems with stochastic nonlinear inverse dynamics and uncertain interactions. [Fan,](#page--1-19) [Han,](#page--1-19) [Wen,](#page--1-19) [and](#page--1-19) [Xu](#page--1-19) [\(2012\)](#page--1-19) first considered the tracking problem for an enlarged class of stochastic interconnected systems.

However, all the existing results are only applicable to systems with interaction effects bounded by static function of subsystem outputs. This condition is restrictive as it is a kind of matching condition in the sense that the effects of all the unmodeled interactions to a local subsystem must be in the range space of the output of this subsystem. In practice, it is unavoidable that an interconnected system has dynamic interactions involving both subsystem inputs and outputs. For example, the non-zero off-diagonal elements of a transfer function matrix represent such interactions.

In this paper we consider the problem of decentralized robust adaptive output-feedback control for a class of stochastic nonlinear systems with dynamic interactions. The main contributions of







this paper are summarized as follows: (1) compared with [Liu](#page--1-18) [et al.](#page--1-18) [\(2007\)](#page--1-18), we consider the more general systems, in which the interactions depend on the subsystem inputs and there exist dynamic interactions and unmodeled dynamics in output channel; (2) the results of both adaptive stabilization in probability and adaptive asymptotic stabilization in probability are obtained; (3) the decentralized adaptive controller design scheme proposed in [Wen](#page--1-3) [et al.](#page--1-3) [\(2009\)](#page--1-3) for deterministic large-scale systems is generalized to stochastic cases.

The organization of this paper is as follows. Section [2](#page-1-0) describes the problem to be investigated and provides some assumptions. The design of decentralized observers is given in Section [3.](#page--1-20) Section [4](#page--1-21) presents the decentralized adaptive control design. The stability analysis of the closed-loop system is provided in Section [5.](#page--1-22) A design example and simulation results are presented in Section [6.](#page--1-23) Conclusions are given in Section [7.](#page--1-24)

**Notations.** For a given vector $x = (x_1, \ldots, x_n)^T$ ,  $|x| = |x_1| +$  $\cdots |x_n|$ . For any matrix  $P \in R^{n \times n}$ ,  $||P|| = \sqrt{Tr(P^T P)}$ , where *Tr*(*P*) denotes the trace of *P*;  $\lambda_{\text{max}}(P)$  stands for the maximal eigenvalue of P. I<sub>i</sub> denotes an identity matrix.  $e_1=(1,0,\ldots,0)^T$ . *K*-function denotes the set of all continuous functions which are strictly increasing and vanishing at zero; *K*∞-function denotes the set of all functions which are class *K*- function and unbounded.

#### <span id="page-1-0"></span>**2. Problem description**

*j*=1

In this paper, a class of stochastic nonlinear large-scale systems is considered, which can be put into a normal form as follows

$$
dx_{i0} = f_{i0}(x_{i0}, y)dt + g_{i0}(x_{i0}, y)dw_i,
$$
  
\n
$$
dx_{i1} = (x_{i2} + f_{i1}(x_{i0}, y))dt + g_{i1}(x_{i0}, y)dw_i,
$$
  
\n
$$
\vdots
$$
  
\n
$$
dx_{i,n_i-1} = (x_{i,n_i} + f_{i,n_i-1}(x_{i0}, y))dt + g_{i,n_i-1}(x_{i0}, y)dw_i,
$$
\n(1)

*j*=1

$$
dx_{i,n_i} = (u_i + f_{i,n_i}(x_{i0}, y))dt + g_{i,n_i}(x_{i0}, y)dw_i,
$$
  

$$
y_i = x_{i1} + \sum_{i}^{N} \mu_{ij}e_1^T p_{ij}(x_{j1}) + \sum_{i}^{N} \nu_{ij}e_1^T q_{ij}(y_j),
$$

 $x_i = [x_{i0}^T, x_{i1}, \dots, x_{i,n_i}]^T$ ,  $u_i \in R, y_i \in R$  (*i* = 1, 2, . . . , *N*) represent the state vector, the scalar control input and the scalar output, respectively;  $y = [y_1, \ldots, y_N]^T$ ;  $x_{i0} \in$ *R mi* is the state of the stochastic inverse dynamics and the initial condition  $x_i(0)$  is fixed; uncertain functions  $f_{ij} \in R, g_{ij}$  $R^{r_i}$  *j* = 0, 1, ...,  $n_i$ , *i* = 1, ..., *N* are locally Lipschitz and locally bounded in their arguments;  $w_i$ ,  $i = 1, ..., N$ , are  $r_i$ dimensional standard Brownian motions defined on a probability space  $(\Omega, F, P)$  with  $\Omega$  being a sample space, *F* being a filtration, and *P* being a probability measure,  $w = (w_1^T, \ldots, w_N^T)^T \in R^r, r =$  $\sum_{i=1}^{N} r_i$ ;  $\mu_{ij}, \nu_{ij}$  are positive constants specifying the magnitudes of dynamic interactions or unmodeled dynamics; *pij*, *qij* are dynamic interactions or unmodeled dynamics, which are generated by

$$
\dot{p}_{ij} = h_{p_{ij}}(p_{ij}, x_{j1}), \n\dot{q}_{ij} = h_{q_{ij}}(q_{ij}, y_j),
$$
\n(2)

where  $h_{p_{ij}}(p_{ij}, x_{j1}), h_{q_{ij}}(q_{ij}, y_j)$  are uncertain functions.

**Remark 1.**  $p_{ij}$  and  $q_{ij}$  denote the dynamic interactions from the state and output of the *j*th subsystem to the *i*th subsystem for  $j \neq i$ , or unmodeled dynamics of the *i*th subsystem for  $j = i$  with  $\mu_{ij}$ and v*ij* indicating the strength of the interactions or unmodeled dynamics. Due to the existence of the dynamic interactions *pij* and *qij*, system [\(1\)](#page-1-1) is more complex and difficult to deal with than that of in [Liu](#page--1-18) [et al.](#page--1-18) [\(2007\)](#page--1-18).

As mentioned above, the objective of this paper is to design a decentralized adaptive output feedback controller for system [\(1\)](#page-1-1) in such a way that the closed-loop system is globally bounded in probability and all the outputs are regulated into a small neighborhood of the origin in probability under some general conditions. To this end, the following assumptions will be imposed on system [\(1\).](#page-1-1)

<span id="page-1-2"></span>**Assumption 1.** For  $x_{i0}$ -subsystem with  $i = 1, 2, ..., N$ , there exist *C*<sup>2</sup> function  $V_{i0}(x_{i0}), K_{\infty}$ -functions  $\alpha_{i1}, \alpha_{i2}, \alpha_{i0}$  and  $\gamma_{i1}, l =$ 1, 2, ..., *N*, such that for any  $(x_{i0}, y) \in R^{m_i} \times R^N$ ,

$$
\alpha_{i1}(|x_{i0}|) \le V_{i0}(x_{i0}) \le \alpha_{i2}(|x_{i0}|),
$$
  
\n
$$
\mathfrak{L}V_{i0}(x_{i0}) \le \sum_{l=1}^{N} \gamma_{il}(|y_l|) - \alpha_{i0}(|x_{i0}|).
$$
\n(3)

**Remark 2.** The definition of the differential operator  $\mathcal{L}V$  can be found in [Deng](#page--1-25) [and](#page--1-25) [Krstic](#page--1-25) [\(2000\)](#page--1-25), [Liu](#page--1-18) [et al.](#page--1-18) [\(2007\)](#page--1-18), [Wang,](#page--1-26) [Chen,](#page--1-26) [and](#page--1-26) [Lin](#page--1-26) [\(2013\)](#page--1-26) and [Wang,](#page--1-27) [Wei,](#page--1-27) [and](#page--1-27) [Wu](#page--1-27) [\(2009\)](#page--1-27).

**Remark 3.** [Assumption 1](#page-1-2) means that the inverse dynamics of the system [\(1\)](#page-1-1) is stochastic input-to-state stable with regard to the input  $y = (y_1, \ldots, y_N)^T$ . The inverse dynamics considered here are nonlinear and enter the subsystems nonadditively.

<span id="page-1-1"></span>**Assumption 2.** For  $i = 1, 2, ..., N$ ,  $j = 1, 2, ..., n$ , there are unknown constants  $a_{ij} > 0$ ,  $b_{ij} > 0$  and known smooth functions  $\varphi_{ij0}$  ≥ 0,  $\varphi_{ijl}$  ≥ 0,  $\psi_{ij0}$  ≥ 0,  $\psi_{ijl}$  ≥ 0, such that for any  $(x_{i0}, y)$  ∈  $R^{m_i} \times R^N$ ,

$$
\left|f_{ij}(x_{i0}, y)\right| \leq a_{ij}\varphi_{ij0}(|x_{i0}|) + a_{ij}\sum_{l=1}^{N}\varphi_{ijl}(|y_l|), \qquad (4)
$$

$$
\left|g_{ij}(x_{i0}, y)\right| \leq b_{ij}\psi_{ij0}(|x_{i0}|) + b_{ij}\sum_{l=1}^{N}\psi_{ijl}(|y_l|). \tag{5}
$$

Without loss of generality, we assume that  $\varphi_{ii0}(0) = 0$  and  $\psi_{ii0}(0) = 0.$ 

**Assumption 3.** Functions  $h_{p_{ij}}(p_{ij}, x_{j1})$  and  $h_{q_{ij}}(q_{ij}, y_j)$  are globally Lipschitz in their variables and vanish at zero. Moreover, the following inequations hold

$$
\left| h_{p_{ij}}(p_{ij}, x_{j1}) \right| \le \rho_{p_{ij}} \left| p_{ij} \right| + \bar{\rho}_{p_{ij}} \left| x_{j1} \right|,
$$
\n
$$
\left| h_{q_{ij}}(q_{ij}, y_{j}) \right| \le \rho_{q_{ij}} \left| q_{ij} \right| + \bar{\rho}_{q_{ij}} \left| y_{j} \right|,
$$
\n(6)

where  $\rho_{p_{ij}}$ ,  $\bar{\rho}_{p_{ij}}$ ,  $\rho_{q_{ij}}$ ,  $\bar{\rho}_{q_{ij}}$  are positive constants.

 $\mathcal{L}$ 

**Assumption 4.** There exist two smooth positive definite radially  $p_{ij}$  and  $\bar{V}_{p_{ij}}$  and  $\bar{V}_{q_{ij}}$  which satisfy  $\underline{d}_{ij}|p_{ij}|^2 \leq \bar{V}_{p_{ij}} \leq \bar{d}_{ij}$  $\left| p_{ij} \right|^2$  and  $\left| \frac{d_{ij}}{dq_{ij}} \right|^2 \leq \bar{V}_{q_{ij}} \leq \bar{d}_{ij} \left| q_{ij} \right|^2$ , such that the following inequations hold.

$$
\frac{\partial \bar{V}_{p_{ij}}}{\partial p_{ij}} h_{p_{ij}}(p_{ij}, 0) \leq -\bar{d}_{p_{ij}1} |p_{ij}|^2, \qquad \left| \frac{\partial \bar{V}_{p_{ij}}}{\partial p_{ij}} \right| \leq \bar{d}_{p_{ij}2} |p_{ij}|,
$$
\n(7a)

$$
\frac{\partial \bar{V}_{q_{ij}}}{\partial q_{ij}} h_{q_{ij}}(q_{ij}, 0) \leq -\bar{d}_{q_{ij}1} |q_{ij}|^2, \qquad \left| \frac{\partial \bar{V}_{q_{ij}}}{\partial q_{ij}} \right| \leq \bar{d}_{q_{ij}2} |q_{ij}|,
$$
 (8a)

where  $\underline{d}_{ij}$ ,  $\overline{d}_{p_{ij}}$ ,  $\overline{d}_{p_{ij}}$ ,  $\overline{d}_{q_{ij}}$ ,  $\overline{d}_{q_{ij}}$  are some positive constants.

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