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A novel hybrid Neumann expansion method for stochastic analysis of mistuned bladed discs

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ABSTRACT

The paper presents a novel hybrid method to enhance the computational efficiency of matrix inversions during the stochastic analysis of mistuned bladed disc systems. The method is based on the use of stochastic Neumann expansion in the frequency domain, coupled with a matrix factorization in the neighbourhood of the resonant frequencies. The number of the expansion terms is used as an indicator to select the matrix inversion technique to be used, without introducing any additional computational cost. The proposed method is validated using two case studies, where the dynamics an aero-engine bladed disc is modelled first using a lumped parameter approach and then with high-fidelity finite element analysis. The frequency responses of the blades are evaluated according to different mistuning patterns via stiffness or mass perturbations under the excitation provided by the engine orders. Results from standard matrix factorization methods are used to benchmark the responses obtained from the proposed hybrid method. Unlike classic Neumann expansion methods, the new technique can effectively update the inversion of an uncertain matrix with no convergence problems during Monte Carlo simulations. The novel hybrid method is more computationally efficient than standard techniques, with no accuracy loss.

1. Introduction

A bladed disk typically consists in a set of disk/blade sectors designed to be identical. However, the identity between sectors is purely theoretical, because uncertainties associated to manufacturing, wear and tear create uneven distributions of the mechanical and geometric properties, both in individual blades and in the whole disk assembly. Dimensional and material uncertainties lead to departures of the natural frequencies of the blades from their nominal design values, creating the so-called "blade mistuning" phenomenon. During free-free vibrations, mistuning separates repeated eigenvalues associated with circumferential modes and distorts the corresponding mode shapes [1–5]. At the same time, the circumferential mode shapes increase the harmonic content of the nodal diameters, leading to the coupling with engine-induced vibrations. In the worst-case scenario, mistuning also causes mode localization phenomena, for which the

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vibrational energy is transferred and confined to only a few blades in the rotor. This may result in dynamic deformations that are significantly larger than those estimated at the design stage [6,7]. Mistuning compromises the high-cycle fatigue resistance of bladed disks, and reduces the endurance and the reliability of the whole engine. It is therefore important to predict accurately the effects of geometric and material uncertainties on the maximum dynamic response in any of the blades of the disk assembly using stochastic analysis (e.g. Monte Carlo simulations, perturbation methods, spectral approach and stochastic reduced basis techniques) [8–10].

Analytical and numerical models of mistuning are generally more cost-effective than direct experimental characterizations [11]. High-fidelity finite element models of bladed disks are therefore commonly used to predict both the maximum dynamic excitation at representative design points and the associated behaviour due to pre-defined mistuning patterns [12,13]. The uncertainty is commonly introduced by using added masses, spring elements or scatter of the material properties of the blades. However, a complete FE model of a bladed disk typically comprises millions of degrees of freedom (DOFs), making stochastic analysis too expensive even using state-of-the-art high-performance computers [14]. The problem is exacerbated by the fact that simplified FE models of single sectors cannot be used, since mistuning breaks the cyclic symmetry of bladed-disk systems. Many reduced order techniques have been proposed to increase the computational efficiency of the stochastic analysis of mistuning. Modal reduction based sub-structuring methods have been extensively developed in the last two decades, with typical examples represented by Component Mode Synthesis (CMS), Fundamental Mistuning Model (FMM), Component Mode Mistuning (CMM), and Integral Mode Mistuning (IMM) [15]. These techniques are able to improve significantly the efficiency of the forced frequency response analysis. Moreover, efficient stochastic simulation techniques have been developed to reduce the number of samples required for the Monte Carlo stochastic analysis, such as importance sampling, accelerated Monte Carlo simulations and Subset Simulation techniques. The latter have proven to yield the best performance so far [16].

For simulations based on stochastic FEMs, the high computational costs of mistuning analyses are mainly due to the repeated inversions of the dynamic stiffness matrix, especially when the scale of the finite element (FE) model is large. However, any direct or reduced method will require the inversion of a dynamic stiffness matrix to obtain the frequency responses of the mistuned systems. Efficient inversion techniques are therefore highly sought after, because the size of the dynamic matrix of bladed disc systems is large, as the scale of the FE model usually involves millions of DOFs. Even reduced order models require considering a large number of modes and interface DOFs. Moreover, thousands of simulations are required in stochastic methods to obtain reliable statistical estimates. Matrix factorization methods, such as the LU decomposition, are commonly used to reduce the computational costs associated with matrix inversions. The main idea behind these techniques is to express the dynamic stiffness as a product between two banded triangular matrices, consisting in permutations of lower and upper triangular matrixes. This way, the dynamic response can be efficiently computed by inverting two banded sub-matrices. The repeated inversions in stochastic analysis can also be potentially avoided by using either the Sherman-Morrison-Woodbury (SMW) formula or the Neumann expansion method (NEM). The SMW formula gives an explicit and efficient expression for the inverse of a matrix perturbed by adding a rank-one update based on the knowledge of the unperturbed inverse matrix (this also called the 'Matrix Inversion Lemma'). The main advantage of using the matrix identity formula is to eliminate expensive repeated inversions in the stochastic analysis, therefore obtaining a significant reduction of the associated computational costs. For this reason, the SMW formula has been applied to the dynamic response of a simple linear system in [17], and also successfully used in the non-linear analysis of systems with frictional dampers [18]. Moreover, this method in principle allows the mistuning analysis of bladed discs via the same FE models that are employed in the analysis of tuned bladed discs via a cyclic symmetry approach [19]. The case study from reference [20] shows that the SMW algorithm can be efficiently used to update the inversion of an uncertain matrix without the need of multiple separated inversions. However, the computational time associated with this modified approach is only 20% smaller when compared to a full LU factorization. The reduction in computational cost depends on the ratio between the numbers of mistuned blades introduced versus the size of the representative model. However, the technique becomes inefficient and even more computationally expensive than LU factorization when more than two blades are perturbed. In this sense, NEM provides an alternative approach to circumvent the direct inversion of an uncertain dynamic stiffness matrix. NEM is based on an iterative process involving the solution of hierarchical linear system of algebraic equations. The NEM technique represents the uncertainty matrix as the sum of deterministic and stochastic matrixes. Neumann series are then used to expand the coefficients of the perturbation matrix, and the solution of the inversion problem is therefore also represented as a series. The random fluctuation range allowed by the NEM is generally larger than the one treatable by other perturbation techniques, because it is easier to incorporate high order terms [21]. However, NEM breaks down when the spectral radius of the iterative matrix is larger than one [22,23]. A scalar-modified NEM approach has been proposed to effectively guarantee convergence with large perturbations [24]. Yuan et.al [20] have investigated the feasibility of using NEMs in stochastic analysis of mistuned bladed disc systems. Results show that NEM can decrease the computational costs by 200% compared with direct matrix factorization approaches, with a maximum relative error smaller than 2%. The convergence of the NEM is however still not guaranteed when the excitation frequency approaches resonance, especially when the system has a low damping and/or a high perturbation level. To some extent, the scalar-modified NEM can reduce the size of the divergence region near resonance, or even eliminate the divergence at low perturbation levels. However, its convergence cannot be completely guaranteed within the practical range of damping and stiffness perturbations routinely used in mistuning analysis.