Automatica 54 (2015) 176–181

Contents lists available at ScienceDirect

Automatica

journal homepage: www.elsevier.com/locate/automatica

Brief paper

Steady-state behaviour of discretized terminal sliding mode ☆

Abhisek K. Behera, Bijnan Bandyopadhyay [1]

Systems and Control Engineering, Indian Institute of Technology Bombay, Mumbai-400 076, India

ARTICLE INFO

Article history:
Received 7 November 2013
Received in revised form
26 October 2014
Accepted 26 January 2015
Available online 24 February 2015

Keywords:
Terminal sliding mode
Discretization
Steady-state behaviour
Periodic orbits

ABSTRACT

In this paper, the steady-state behaviour of discretized terminal sliding mode control is studied. A discrete terminal sliding mode control is designed by discretizing the continuous-time system and then the steady-state behaviour is analysed in terms of periodic orbits. We have shown that by this discrete terminal sliding mode control the discretized second order system exhibits only period-2 motion in steady-state. To ensure this, existence and stability conditions of all possible periodic orbits are found out analytically and shown that only period-2 conditions are satisfied by the state evolution of the discretized system in steady-state. Next, a higher order single-input-single-output system is considered and we also establish that only period-2 motion exists in steady-state.

1. Introduction

Sliding mode control (SMC) is one of the robust control techniques that has drawn interest among control communities since the last four decades due to its ability to reject the disturbance completely, order reduction, invariance property etc. (Draženović, 1969; Utkin, 1977). The other advantages of the SMC are simplicity in design and its implementation. In the early nineties, a new concept on sliding mode was reported based on the terminal attractor called the terminal sliding mode (TSM) controller (Venkataraman & Gulati, 1993). In addition to the above properties, TSM controller guarantees finite time stability of the closed loop system. The central idea of the controller is to design a nonlinear sliding surface so that finite time convergence of the system is achieved once the sliding mode is enforced. Since then many developments in TSM controller have been reported (Chiu, 2012; Huang, Lin, & Yang, 2005; Man, Paplinski, & Yu, 1994; Man & Yu, 1997; Yu & Man, 1996). Due to these properties, the SMC and TSM have been implemented successfully in many practical systems and few of them can be found in Abidi, Xin, and She (2009), Feng, Zheng, Yu, and Truong (2009), Li, Zhou, and Yu (2013) and Utkin, Guldner, and Shi (1999). In practical applications, almost all the controllers are implemented digitally. The availability of high performance digital processors make it possible to implement the continuous controllers digitally without deteriorating the system response. However, in the systems where control is implemented digitally not fast enough compared to continuous-time analogy, the discrete-time design becomes relevant. The discrete-time sliding mode control (DTSM) is not able to make the system slide on the surface but within a band called quasi sliding mode band (QSM) (Gao, Wang, & Homaifa, 1995; Sarpturk, Istefanopulos, & Kaynak, 1987).

In very few papers, the practical implementation issues of TSM have been addressed. For the first time discretization of TSM was studied in Janardhanan and Bandyopadhyay (2006). It has been shown that the system states never go to zero and it results a periodic behaviour in the system trajectories in steady-state. Moreover, stability of only period-2 orbit is reported and no further higher orbits are investigated. Recently, in Galias and Yu (2012) and Li, Du, and Yu (2014), discretization issues of TSM controller is studied by discretizing the continuous control law. In Galias and Yu (2012), authors studied both period-2 and period-4 orbits for the closed-loop system stability for second order system only. However, no evidence is given for existence of higher orbits which may affect the stability of the system. The extended version of discretization of TSM control for a nth order SISO system is reported in Li et al. (2014).

1.1. Motivation

Mainly two approaches are used to implement the controller in practical systems. First, continuous controller is designed for continuous-time system and then it is implemented in real plant

☆ The material in this paper was not presented at any conference. This paper was recommended for publication in revised form by Associate Editor Zhihua Qu under the direction of Editor Andrew R. Teel.

E-mail addresses: abhisek@sc.iitb.ac.in (A. Behera), bijnan@ee.iitb.ac.in (B. Bandyopadhyay).

[1] Tel.: +91 22 2576 7889; fax: +91 22 2570 0057.

http://dx.doi.org/10.1016/j.automatica.2015.02.009
0005-1098/

in discrete manner. However, simply discretization of the continuous control never achieves desired objective if the control is not applied very fast. The results in Galias and Yu (2012) and Li et al. (2014) are based on this approach. Secondly, design the discrete controller by discretizing the continuous-time system and then apply it at discrete instants. This technique is more effective as the controller is applied at discrete instants only. This approach requires discretized model of continuous-time nonlinear system. So, in this paper, we have adopted this methodology to design the discrete TSM control. Under this TSM control, the stability of the system and TSM are analysed and steady-state behaviour is addressed in terms of periodic orbits.

1.2. Main contribution

The main contributions in this paper are: (1) Analyse the behaviour of the system completely when the system slides. All possible periodic orbits have been analysed analytically and their existence conditions are given which have not been reported earlier. (2) The steady-state bounds for the discretized system are obtained which are functions of sampling step between two consecutive time instants of discrete control law. Further, we establish that in steady-state only period-2 orbit is exhibited by the system.

The rest of the paper is organized as follows. Section 2 briefly reviews the continuous-time TSM control. The discretization of continuous-time TSM is presented in Section 3. We find analytically all possible periodic orbits. The stability analysis of periodic solutions are carried out in Section 4. The steady-state behaviour of both second and higher order system are analysed separately. Some concluding remarks are drawn in Section 5.

Notation. The $\mathbb{R}$ and $\mathbb{R}^n$ denote the set of all real numbers and n-dimensional linear space, respectively. The $\mathbb{Z}_{\geqslant 0}$ denotes the set of all nonnegative integers. For any functions $f(x)$ and $g(x)$, we denote $f \circ g(x) \triangleq f(g(x))$. Similarly, $f^n(x)$ is defined as nth iteration of $f(x)$, i.e., $f^n(x) \triangleq \overbrace{f \circ f \circ \cdots \circ f}^{n\text{ times}}(x) = f(f(\cdots f(x)\cdots))$. For discrete-time systems, the value of any function at kth instant $f(kh)$ is denoted as $f(k)$ where h is the sampling interval. We define $\Delta f(n) \triangleq f(n+1) - f(n)$ for any $n \in \mathbb{Z}_{\geqslant 0}$. The notation f^+ is defined as forward difference operator and is given as $f^+ := \frac{\Delta f(k)}{h} = \frac{f(k+1)-f(k)}{h}$ for any given $h > 0$. Throughout the paper the notation $x_j^{(i)}$ corresponds to i^{th} periodic point of period-j where $i = 1, 2, \ldots, j$.

2. Continuous-time TSM control

Consider the following second order system

$$\begin{cases}\dot{x}_1 = x_2\\ \dot{x}_2 = f(x) + g(x)u\end{cases} \tag{1}$$

where $x_i \in \mathbb{R}$ for $i = 1, 2$, and $f(\cdot)$ and $g(\cdot)$ are continuous functions. We assume $g(\cdot) \neq 0$ for every $x \in \mathbb{R}^2$. The terminal sliding surface is defined as (Venkataraman & Gulati, 1993)

$$s = x_2 + \beta x_1^{\eta} \tag{2}$$

where $\beta > 0$ and η is selected as the ratio of two positive odd positive integers represented as $\eta = q/p$ with $0 < q < p$. Its value is selected as $\eta \in [0.5, 1)$ to obtain the bounded control during sliding. The control law to bring TSM is given as

$$u = -g^{-1}(x)\left(f(x) + \beta\eta x_1^{\eta-1}x_2 + K\,\mathrm{sign}(s)\right) \tag{3}$$

where $K > 0$. Once the sliding surface is reached, the states of the system go to zero in finite time and is calculated as $t_1 = T + \frac{x_1^{1-\eta}(T)}{(1-\eta)\beta}$ where T denotes the time taken by the system to reach the sliding surface. Consider a higher order SISO system as

$$\begin{cases}\dot{x}_i = x_{i+1}, \quad i = 1, \ldots, n-1\\ \dot{x}_n = f(x) + g(x)u\end{cases} \tag{4}$$

where $x = [x_1\, x_2\, \cdots\, x_n]^{\top} \in \mathbb{R}^n$ and $f(x)$ and $g(x)$ are defined as earlier. The TSM manifold for the system (4) are defined in nested form as

$$s_i = \dot{s}_{i-1} + \beta_i s_{i-1}^{\eta_i}, \quad i = 1, \ldots, n-1. \tag{5}$$

The constants $\beta_i > 0$ for all $i = 1, \ldots, n-1$ with $s_0 = x_1$ and the exponents η_i are ratios of positive odd integers chosen such that control u remains bounded during sliding (Yu & Man, 1996). The TSM control u is designed such that the sliding variable s_{n-1} becomes zero in finite time. This further leads to s_{n-2} and finally s_0 to zero subsequently. As $s_0 = x_1$ becomes zero, the finite time convergence of the system (4) is achieved. The control law expression which leads the TSM is given as

$$u = -g^{-1}(x)\left(f(x) + \sum_{i=1}^{n-1}\frac{\mathrm{d}^i}{\mathrm{d}t^i}\beta_{n-i}s_{n-i-1}^{\eta_{n-i}} + K\,\mathrm{sign}(s_{n-1})\right) \tag{6}$$

where $K > 0$ is needed to ensure finite time convergence.

3. Discretization of continuous-time TSM

In this section, we consider the second order system (1). The Euler discretization is used throughout the paper for its simplicity. Let us consider system (1) in discrete domain

$$\begin{cases}x_1(k+1) = x_1(k) + hx_2(k)\\ x_2(k+1) = x_2(k) + hf(x(k)) + hg(x(k))u(k)\end{cases} \tag{7}$$

where h is the sampling time of the Euler discretization. Similarly, the discrete equivalent of sliding surface is given as

$$s(k) = x_2(k) + \beta x_1^{\eta}(k). \tag{8}$$

A discrete equivalent control law $u(k)$ can be derived using the sliding surface (8) and system dynamics (7) which always brings the trajectory of the system exactly onto the sliding surface in one sampling instant and for all $k \in \mathbb{Z}_{\geqslant 0}$ maintains $s(k+1) = 0$. The controller which brings $s(k)$ to zero is given as

$$u(k) = -(hg(x(k)))^{-1}(x_2(k) + hf(x(k)) + \beta(x_1(k) + hx_2(k))^{\eta}). \tag{9}$$

As soon as the sliding surface is reached, the discrete dynamics is governed by the dynamics of $x_1(k)$ only. The dynamics of the system on the terminal sliding surface can then be given as

$$\Phi(x_1) = x_1(k+1) = x_1(k) - h\beta x_1^{\eta}(k). \tag{10}$$

Remark 1. The control (9) uses only the state information at some discrete sampling instants and is different from (3). This control law achieves sliding mode, i.e., $s(k) = 0$ like in continuous-time controller. Moreover, the control (3) when implemented digitally never achieves $s(k) = 0$.

Remark 2. The behaviour of the system (7) on sliding surface (8) is now reduced to (10). So, the behaviour of system (7) depends on (10). In other words, analysing the steady-state behaviour of (10) can give steady-state behaviour of closed-loop system.

The steady-state solutions of the system (10) are found to be periodic orbits as reported in Galias and Yu (2012) and Janardhanan and Bandyopadhyay (2006). Here, we find all possible periodic orbits of the system and show the existence of an orbit in steady-state for any given values of h, β and η. The rest of the analysis follows to find the steady-state orbit.