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Quadratic partial eigenvalue assignment in large-scale stochastic dynamic systems for resilient and economic design

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ABSTRACT

Failure of structural systems under dynamic loading can be prevented via active vibration control which shifts the damped natural frequencies of the systems away from the dominant range of a loading spectrum. The damped natural frequencies and the dynamic load typically show significant variations in practice. A computationally efficient methodology based on quadratic partial eigenvalue assignment technique and optimization under uncertainty has been formulated in the present work that will rigorously account for these variations and result in economic and resilient design of structures. A novel scheme based on hierarchical clustering and importance sampling is also developed in this work for accurate and efficient estimation of probability of failure to guarantee the desired resilience level of the designed system. Finally the most robust set of feedback matrices is selected from the set of probabilistically characterized optimal closed-loop system to implement the new methodology for design of active controlled structures. Numerical examples are presented to illustrate the proposed methodology.

1. Introduction

Infrastructures (e.g., high-rise buildings, bridges, wind-turbines), automotive and mechanical devices (e.g., aircrafts, hard disk drives), etc., are subjected to various types of dynamic loads during their service life. They undergo vibration with high amplitude (resonance phenomenon) when some of their damped natural frequencies are close to the dominant frequencies of the dynamic loads. If the design process for such systems fails to properly account for the effects of the dynamic loads on the response of the systems, the designed structure may collapse (e.g., failure of the Tacoma Narrows bridge, Washington in 1940 [1,2]) or lose functionality (e.g., wobbling of the Millennium bridge, London in 2000 [3,4]). The catastrophic consequences can be prevented through deployment of appropriate vibration control strategies, among which the most popular are the active vibration control (AVC) and the passive vibration control (PVC). The AVC is, however, advantageous over the PVC due to its ability to reduce the vibration level of a structure by modifying only those damped natural frequencies of the structure that lie within the dominant spectrum of the loads. A schematic of the AVC technique and various components of an AVC system employed on a high-rise building are depicted in Fig. 1.

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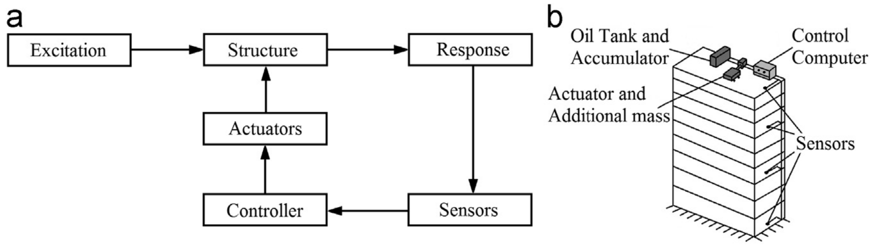

Fig. 1. Schematic of (a) AVC technique and (b) various components of an AVC system.

To understand the working principle of AVC technique, let us consider the governing equation representing the dynamics of structures given by

$$\mathbf{M}\ddot{\mathbf{q}}(t)+\mathbf{C}\dot{\mathbf{q}}(t)+\mathbf{K}\mathbf{q}(t)=\mathbf{f}(t) \tag{1}$$

Eq. (1) is obtained via the finite element (FE) discretization of the governing partial differential equations (PDEs) approximating the dynamic response of the physical structural system. In Eq. (1), $\mathbf{M},\mathbf{C},\mathbf{K}$ denote the mass, damping, and stiffness matrices of the system, respectively. Also, $\mathbf{q}(t)$ denotes the displacement response of the system, $\mathbf{f}(t)$ denotes the dynamic load acting on the system and the dot $\dot{()}$ denotes time derivative. Note that $\mathbf{M},\mathbf{C},\mathbf{K}\in\mathbb{M}_n^+(\mathbb{R})$, the set of all real symmetric positive-definite matrices of size $n\times n$ where n is the degrees of freedom (dof) associated with the FE model of the system. Since $\mathbf{M},\mathbf{C},\mathbf{K}$ are generated by FE technique, they are very large and often inherit nice structural properties, such as definiteness, bandness, and sparsity, which are useful for large-scale computing. The system is, in general, denoted by $(\mathbf{M},\mathbf{C},\mathbf{K})$ and is known as open-loop system. The dynamics of $(\mathbf{M},\mathbf{C},\mathbf{K})$ depend on their damped natural frequencies ω_{dn} and mode-shapes $\mathbf{x}_{dn}$. The pair $(\omega_{dn},\mathbf{x}_{dn})$ of the system can be found by solving quadratic eigenvalue problem (QEP) associated with the quadratic pencil $\mathbf{P}_o(\lambda)=\left(\mathbf{M}\lambda^2+\mathbf{C}\lambda+\mathbf{K}\right)$, where λ denotes the eigenvalue of the QEP given by $\mathbf{P}_o(\lambda)\mathbf{x}=\mathbf{0}$. Here $\mathbf{x}$ denotes the right eigenvector of the QEP and is associated with the eigenvalue λ. Note that λ's occur in complex conjugate pairs and $\omega_{dn}=\mathrm{Im}(\lambda)$.

The mathematical basis of AVC is an inverse eigenvalue problem for the pencil $\mathbf{P}_o(\lambda)$, known as the quadratic partial eigenvalue assignment (QPEVA) problem. Suppose that a control force of the form $\mathbf{B}\mathbf{u}(t)$ is applied to the structure for vibration control, where $\mathbf{B}\in\mathbb{M}_{n\times m}(\mathbb{R})$ with $m\le n$ is a fixed control matrix and the control input vector $\mathbf{u}(t)$ assumes the form $\mathbf{u}(t)=\mathbf{F}^T\dot{\mathbf{q}}(t)+\mathbf{G}^T\mathbf{q}(t)$. The matrices $\mathbf{F},\mathbf{G}\in\mathbb{M}_{n\times m}(\mathbb{R})$ are feedback gain matrices. Under the application of control force, Eq. (1) then gets modified to

$$\mathbf{M}\ddot{\mathbf{q}}(t)+\left(\mathbf{C}-\mathbf{B}\mathbf{F}^T\right)\dot{\mathbf{q}}(t)+\left(\mathbf{K}-\mathbf{B}\mathbf{G}^T\right)\mathbf{q}(t)=\mathbf{f}(t) \tag{2}$$

The modified system is called a closed-loop system and is denoted by $\left(\mathbf{M},\left(\mathbf{C}-\mathbf{B}\mathbf{F}^T\right),\left(\mathbf{K}-\mathbf{B}\mathbf{G}^T\right)\right)$. The dynamics of this system are then governed by the eigenvalues and eigenvectors of the closed-loop pencil $\mathbf{P}_c(\lambda)=\left(\mathbf{M}\lambda^2+\left(\mathbf{C}-\mathbf{B}\mathbf{F}^T\right)\lambda+\left(\mathbf{K}-\mathbf{B}\mathbf{G}^T\right)\right)$.

Let the set of eigenvalues of the $(\mathbf{M},\mathbf{C},\mathbf{K})$ be denoted as $\lambda_i{}_{i=1}^{2n}$ with $\lambda_i{}_{i=1}^{2p}$ (with $2p\ll 2n$) representing the set of problematic eigenvalues which needs to be replaced by $\mu_i{}_{i=1}^{2p}$. Let $\mathbf{x}^{(i)}_{i=1}2p$ be the right eigenvectors of the $(\mathbf{M},\mathbf{C},\mathbf{K})$ associated with the eigenvalues $\lambda_i{}_{i=1}^{2p}$. Then QPEVA problem is to find feedback matrices $\mathbf{F}$ and $\mathbf{G}$ such that the spectrum of $\mathbf{P}_c(\lambda)$ is $\{\mu_i{}_{i=1}^{2p},\lambda_i{}_{i=2p+1}^{2n}\}$ and the eigenvectors $\mathbf{x}^{(i)}_{i=2p+1}2n$ corresponding to $\lambda_i{}_{i=2p+1}^{2n}$ remain unchanged. This desired feature is known as *no-spillover property*. In this context, note that, the QPEVA technique is computationally different from the traditional pole/eigenvalue assignment technique for which there exist excellent numerically viable methods [5, Sections 11.2–11.3].

In the past, a handful number of algorithms based on the state-space approach have been developed to implement QPEVA technique for AVC [6–11]. Also, in the last two decades, considerable research efforts have been channelized towards implementation of state-space approach based AVC techniques for buildings [12–17], wind turbines [18–23], hard disk drives [24–26], etc. These state-space approach based AVC techniques, however, turn out to be numerically inefficient for large-scale systems as the matrices associated with the linear model are twice the dimension of $\mathbf{M},\mathbf{C},\mathbf{K}$ and they lose the inherent computationally exploitable properties stated earlier. In this regard, the mathematical and engineering challenges are to solve the QPEVA problem in the quadratic setting itself and using only a small number of eigenvalues and eigenvectors that are all that can be computed using the state-of-the-art computational techniques (e.g., Jacobi–Davidson methods [27,28]) or can be measured in a vibration laboratory [29–31]. Furthermore, the no-spillover property must be established by means of mathematical results, because it is not possible to do so computationally or by measurements. Meeting these challenges, several computationally effective algorithms have been developed in the recent years for large-scale systems [32–34]. Furthermore, work has been done to compute the feedback matrices such that they have minimum norm (MNQPEVA technique) and the robustness of the closed-loop system under small perturbations of the data is ensured (RQPEVA technique). These algorithms are very well suited in a deterministic setting.