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Brief paper

Containment control of heterogeneous linear multi-agent systems[☆]

Hamed Haghshenas, Mohammad Ali Badamchizadeh[1], Mahdi Baradarannia

Department of Control Engineering, Faculty of Electrical and Computer Engineering, University of Tabriz, Tabriz, Iran

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ABSTRACT

In this note, we study the containment control problem of heterogeneous linear multi-agent systems based on output regulation framework. Motivated by leader–follower output regulation problems, the leaders are assumed to be exosystems. In controller design approach for each follower, we utilize a distributed dynamic state feedback control scheme. To achieve the objective of this work, we modify the conventional output regulation error in such a way that it can handle more than one leader, and we also introduce a dynamic compensator. Our work is based on a new formulation for containment error that guarantees the convergence of all follower agents to the dynamic convex hull spanned by the leaders, and also enables us to use output regulation techniques with some modifications to solve the containment problem. Finally, a numerical example is given to illustrate the validity of theoretical results.

1. Introduction

Recently, a large body of work has emerged on the topic of distributed coordination of multi-agent systems due to its broad applications in spacecraft formation flying, sensor networks, cooperative surveillance, and so forth (Olfati-Saber, Fax, & Murray, 2007; Ren, Beard, & Atkins, 2007). Consensus has gained a great attention among the researchers in recent years as one of the most ubiquitous phenomena in the area of cooperative control of multi-agent systems. Consensus problems were primarily concentrated on the leaderless systems, such as works presented in Olfati-Saber et al. (2007) and Ren et al. (2007) and references therein. By considering that a group of agents might have a desired reference trajectory, consensus problem for the leader-following cooperative systems has also been studied widely, such as in Yang and Peng (2009).

Recently, cooperative output regulation of multi-agent systems has been a topic in consensus problems. A leaderless output consensus problem of heterogeneous linear multi-agent systems based on internal model principle has been studied in Wieland, Sepulchre, and Allgöwer (2011). Cooperative output regulation of leader–follower multi-agent systems, by treating a leader as an exosystem has been discussed in the literature; for instance see Hong, Wang, and Jiang (2013) and Su and Huang (2012a). In Su and Huang (2012a), the problem has been solved by introducing a dynamic compensator and a dynamic full information distributed control scheme for linear systems. The authors in Hong et al. (2013) have discussed the problem for some classes of multi-agent systems with switching topologies via both static and dynamic feedback. Also, cooperative output regulation of nonlinear systems has been studied in recent years. In Su and Huang (2013), this problem has been investigated for heterogeneous second-order nonlinear uncertain multi-agent systems.

Containment, as a particular subarea of multi-agent systems control, has been investigated a lot. Containment control problems are motivated by some natural phenomena and have potential and vital applications in practice. One application of containment is to ensure that a group of robots or autonomous vehicles does not venture into hazardous areas. In this case, a portion of agents are introduced as leaders to make the vehicles or robots move into the safe region spanned by the leaders.

Most of the existing works on the containment problem have considered the case of homogeneous multi-agent systems, in which all the agents have identical dynamics. In real world applications, a network of agents with different dynamics and abilities are more applicable than homogeneous systems. Also, because of various restrictions in the practical systems, the agents are governed by different dynamics with each other. In Lou and Hong (2010), the followers are governed by double-integrator dynamics, but the leader agents are supposed to have single-integrator dynamics. Recent work (Zheng & Wang, 2014) has investigated the containment problem for multi-agent systems with heterogeneous agents

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E-mail addresses: haghshenas91@ms.tabrizu.ac.ir (H. Haghshenas), mbadamchi@tabrizu.ac.ir (M.A. Badamchizadeh), mbaradaran@tabrizu.ac.ir (M. Baradarannia).

[1] Tel.: +98 411 3393729; fax: +98 411 3300819.

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by considering two cases: in the first case, the authors have presented a linear protocol where the dynamics of the leaders and followers are assumed to be first-order and second-order integrators, respectively. In the second case, the leaders and followers are assumed to have second-order and first-order integrator dynamics, respectively. In this case, a nonlinear protocol has been proposed to solve the containment problem in finite-time. The containment control problem has been studied recently in Haghshenas, Badamchizadeh, and Baradarannia (submitted for publication) for a group of non-identical agents, where the dynamics of agents are supposed to be nonlinear with unknown parameters and parameterized by some basis functions. In controller design approach for each follower, adaptive control and Lyapunov theory are utilized as the main control strategies. Because of utilizing adaptive control in Haghshenas et al. (submitted for publication), there exists an assumption on the corresponding graphs which limits the range of communication graphs.

The purpose of this paper is to study the containment control problem for heterogeneous linear multi-agent systems in cooperative output regulation framework. Motivated by leader–follower output regulation problems, we consider the leaders to be exosystems. Compared to Lou and Hong (2010) and Zheng and Wang (2014) where the follower and leader agents have different dynamics but the dynamics between each of two followers are the same, in this paper, we assume that the dynamics of all followers are non-identical. The main contributions of this work are twofold. First, we present a design approach for containment control in output regulation framework by modifying the conventional output regulation error in such a way that it can handle more than one leader. We also propose a dynamic compensator and a distributed dynamic state feedback control law. Second, our analysis uses a new formulation for containment error, in which the containment error is formulated using M-matrices H_k defined in (7), which enables us to use output regulation techniques with some modifications to solve the containment problem.

The subsequent sections are organized as follows: some basic concepts of graph theory, definitions and notations are presented in Section 2. Section 3 formulates the model to be studied and states the problem. The main results of this work are presented in Section 4. Numerical simulation is carried out in Section 5 to validate the effectiveness of the proposed approach, and finally, Section 6 concludes the whole paper.

2. Preliminaries

2.1. Graph theory

Graphs are used to model the interaction among the agents in multi-agent systems. For a group of n agents, the corresponding directed or undirected graph $\mathcal{G}$ can be expressed by a pair $(\mathcal{V}, \mathcal{E})$, where $\mathcal{V} = \{1, 2, \ldots, n\}$ is the set of nodes and $\mathcal{E} \subseteq \mathcal{V} \times \mathcal{V}$ is the set of edges. An edge denoted as $(i, j) \in \mathcal{E}$ means that agent j has access to the information of agent i and also, agent i is a neighbor of agent j. Graph $\mathcal{G}$ is said to be undirected if $(i, j) \in \mathcal{E}$ implies $(j, i) \in \mathcal{E}$. A directed path from node i to node j is a sequence of nodes starting with i and ending with j such that consecutive nodes are adjacent in the form $(i, p), (p, q), \ldots, (r, j)$. A directed graph is called strongly connected, if for any ordered pair of distinct nodes $[i, j]$, there is a directed path from node i to node j.

The adjacency matrix $\mathcal{A} = [a_{ij}] \in \mathbb{R}^{n\times n}$ associated with $\mathcal{G}$ is defined such that $a_{ij} > 0$ if $(j, i) \in \mathcal{E}$, $a_{ii} = 0$, and $a_{ij} = 0$ otherwise. The Laplacian matrix $\mathcal{L} = [l_{ij}] \in \mathbb{R}^{n\times n}$ of graph $\mathcal{G}$ is defined as $l_{ii} = \sum_{j=1,j\neq i}^{n} a_{ij}$ and $l_{ij} = -a_{ij}$, $i \neq j$.

In this note, we consider a group of $n + m$ agents, composed of n followers and m leaders. Let $\mathcal{G}_{n+m}$ and $\mathcal{G}_n$ be the graphs used to model $n + m$ agents, and n followers interaction topology, respectively. $\mathcal{G}_n$ can be obtained from $\mathcal{G}_{n+m}$ by removing the leaders nodes and edges connected to them. In the subsequent sections, we denote the adjacency matrix and Laplacian matrix associated with $\mathcal{G}_n$ by $\mathcal{A}$ and $\mathcal{L}$, respectively. For the follower and leader agents, we use indices $\{1, \ldots, n\}$ and $\{n + 1, \ldots, n + m\}$, respectively. Also, $\mathcal{N}_i$ is used to denote the set of neighbors of follower i with respect to the follower set.

The connection weight of leader k is denoted by Δ_k, which is a diagonal matrix with diagonal elements $\delta_1^k, \delta_2^k, \ldots, \delta_n^k$, where δ_i^k is defined to be 1 if the ith follower is connected to the kth leader, and 0 otherwise.

2.2. Definitions and notations

Throughout this note, we denote the set of leaders and the set of followers by $\mathcal{R}$ and $\mathcal{F}$, respectively. Let $\mathbf{1}_n$, I_p and $\mathbf{0}$ be the $n \times 1$ column vector of all ones, the identity matrix of order p, and the zero matrix with appropriate dimension, respectively. A block-diagonal matrix formed by $x_1, x_2, \ldots, x_n$ is denoted by diag $\{x_1, x_2, \ldots, x_n\}$. Symbol $\otimes$ represents the Kronecker product. Denote by dist $(x, \mathcal{C})$ the distance from $x \in \mathbb{R}^N$ to the set $\mathcal{C} \subseteq \mathbb{R}^N$ in the sense of Euclidean norm, that is

$$\mathrm{dist}(x, \mathcal{C}) = \inf_{y\in\mathcal{C}} \|x - y\|_2. \tag{1}$$

Definition 1 (*Rockafellar, 1972*)**.** A set $\mathcal{C} \subseteq \mathbb{R}^N$ is convex if $(1 - \lambda)x + \lambda y \in \mathcal{C}$, for any $x, y \in \mathcal{C}$ and any $\lambda \in [0, 1]$. The convex hull $\mathrm{Co}(X)$ of a finite set of points $X = \{x_1, x_2, \ldots, x_q\}$ is the minimal convex set containing all points in X. That is, $\mathrm{Co}(X) = \{\sum_{i=1}^{q} \alpha_i x_i | x_i \in X, \alpha_i \in \mathbb{R}, \alpha_i \geq 0, \sum_{i=1}^{q} \alpha_i = 1\}$.

3. Problem formulation

The multi-agent system considered here is composed of n non-identical followers with linear dynamics and m identical leaders. The dynamics of the ith follower are given by

$$\dot{x}_i = A_i x_i + B_i u_i, \quad i \in \mathcal{F}, \tag{2}$$

where $x_i \in \mathbb{R}^N$ and $u_i \in \mathbb{R}^p$ are the state and control input of the ith follower, respectively, and A_i, B_i are constant matrices with compatible dimensions. The m leaders are assumed to be exosystems as follows:

$$\dot{w}_k = S w_k, \quad k \in \mathcal{R}, \tag{3}$$

where $w_k \in \mathbb{R}^N$ is the exogenous state of the kth leader.

Motivated by the output regulation of leader–follower multi-agent systems, we address the containment control problem of multi-agent system (2)–(3), defined as follows.

Definition 2. The multi-agent system (2)–(3) achieves containment if the control law for each follower can be designed to make sure that all followers will converge to the convex hull spanned by the dynamic leaders as $t \to \infty$, that is $\forall i \in \mathcal{F}$

$$\lim_{t\to\infty} \mathrm{dist}\big(x_i(t), \mathrm{Co}(w_k(t), k \in \mathcal{R})\big) = 0. \tag{4}$$

We begin by introducing the error vector of the ith follower as

$$e_i(t) = \sum_{j\in\mathcal{N}_i} a_{ij}(x_i - x_j) + \sum_{k=n+1}^{n+m} \delta_i^k (x_i - w_k), \quad i \in \mathcal{F}. \tag{5}$$

The overall error vector can be written as

$$e(t) = \sum_{k=n+1}^{n+m} (H_k \otimes I_N)(x - \mathbf{1}_n \otimes w_k(t)), \tag{6}$$