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Brief paper

Switched adaptive control of switched nonlinearly parameterized systems with unstable subsystems☆

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ABSTRACT

This paper investigates the problem of adaptive stabilization for a class of switched nonlinearly parameterized systems where the solvability of the adaptive stabilization problem for subsystems is unnecessary. A new switched adaptive control technique for the problem studied is set up by exploiting the generalized multiple Lyapunov functions method and the parameter separation technique. Different update laws are designed to reduce the conservativeness caused by adoption of a common update law for all subsystems. Also, the proposed technique permits removal of a common restriction in which the parameterization in the switched systems is restricted to a linear parameterization. As an application of the proposed design technique, stabilization for a class of non-triangular switched systems with nonlinear parameterization is achieved by design of adaptive controllers. Since no subsystem of such a switched system is assumed to be adaptively stabilizable, the problem under study cannot be handled by the existing methods. A two inverted pendulums as a practical example is also provided to demonstrate the effectiveness of the proposed design method.

1. Introduction

Adaptive control of non-switched nonlinear systems with parametric uncertainty has attracted considerable attention in the field of nonlinear control (Astolfi & Ortega, 2003; Huang, 2009; Karagiannis & Astolfi, 2008; Krstic, Kanellakopoulos, & Kokotovic, 1995; Tao, 2003). One of the reasons for the rapid growth and continuing popularity of adaptive control is its clearly defined goal: to control systems with uncertainties by estimating unknown system's parameters. For about two decades, there is a vast amount of literature on design and analysis of various adaptive control systems using rigorous proofs (Back, Cheong, Shim, & Seo, 2007; Haddad, Hayakawa, & Chellaboina, 2003; Hong, Wang, & Cheng, 2009; Spooner & Passino, 1999). In particular, adaptive control has proven its great capability in compensating for non-switched nonlinearly parameterized systems involving inherent nonlinearity on the basis of a parameter separation technique (Lin & Qian, 2002a,b).

On the other hand, the study of switched systems has been extensively investigated in the last decade (Cao & Morse, 2010; Colaneri, Geromel, & Astolfi, 2008; Girard, Pola, & Tabuada, 2010; Goebel, Sanfelice, & Teel, 2009; Mancilla-Aguilar & García, 2006; Qiao & Cheng, 2009), and the rapidly developing area of intelligent control, such as robotic, mechatronic and mechanical systems, gene regulatory networks, switching power converters, is an important source of motivation for this study (Hespanha, 2003; Lin & Antsaklis, 2009; Mojica-Nava, Quijano, Rakoto-Ravalontsalama, & Gauthier, 2010; Serres, Vivalda, & Riedinger, 2011). Meanwhile, many methodologies such as single Lyapunov function, multiple Lyapunov functions (MLFs), average dwell-time have been proposed based on some specified classes of switching laws in the study of switched systems (Branicky, 1998; Han, Ge, & Lee, 2010; Liberzon, 2003; Liberzon & Morse, 1999; Long & Zhao, 2012, 2014b).

Recently, some results on adaptive control for switched linear systems have appeared (see, e.g., Bernardo, Montanaro, & Santini, 2008, Chiang & Fu, 2009, Di Bernardo, Montanaro, & Santini, 2013,

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Khalid, Osamah, & Kamal, 2005, Sang & Tao, 2012). However, there has been relatively little work for switched nonlinear systems with parametric uncertainty up to now. Adaptive stabilization for strict-feedback switched nonlinear systems under arbitrary switchings is achieved via backstepping which provides a common Lyapunov function (CLF) (Tamba & Leksono, 2010); in Han, Ge, and Lee (2009), an adaptive NN control scheme for strict-feedback switched nonlinear systems is proposed for switchings with a certain dwell-time; When the solvability of the disturbance rejection problem for subsystems is not assumed, an adaptive disturbance rejection problem for switched nonlinear systems in strict-feedback form with unknown exosystem is studied in Long and Zhao (2014a). It should be observed that the literature mentioned above has focused on adaptive control of switched nonlinear systems with linear parameterization, and a common update law is used to estimate all the vector parameters in different subsystems. To the best of our knowledge, no results in the switched nonlinearly parameterized systems have been reported. The crucial obstacle in the study is the complexity arising from the interaction among the system structure, uncertain parameters and switchings. There are two main issues to be addressed: When no adaptive stabilization problem for subsystems is solvable, how to solve the adaptive stabilization problem via design of a switching law and adaptive state-feedback controllers? In particular, how to design different update laws and a MLFs based switching signal to reduce the conservativeness caused by adoption of a common update law for all subsystems? Hence, the study of adaptive control of switched nonlinearly parameterized systems is of great significance and remains an open area.

Motivated by the above considerations, this paper studies the adaptive control problem for switched nonlinearly parameterized systems. None of individual subsystems is assumed to be globally adaptively stabilizable. Compared with the vast existing literature on switched nonlinear systems, the results of this paper have three distinct features. First of all, a main result about adaptive control for switched nonlinearly parameterized systems is established for the first time, which provides a tool for analyzing the behavior of switched nonlinearly parameterized systems. A sufficient condition for the adaptive stabilization problem is derived by exploiting the generalized multiple Lyapunov functions (GMLFs) method (Zhao & Hill, 2008) and the parameter separation technique (Lin & Qian, 2002a). Secondly, in order to reduce the conservativeness caused by adoption of a common update law for all subsystems, different update laws are designed. Also, the switched adaptive control technique permits removal of a common restriction in which the parametric uncertainty in the switched systems is restricted to a linear parameterization. Finally, an application of the design procedure to non-triangular switched systems with nonlinear parameterization is investigated. The dual design of adaptive controllers and switching laws are constructive by designing the MLFs and different update laws of all subsystems based on the GMLFs method and the parameter separation technique and the adding a power integrator technique (Lin & Qian, 2002a; Qian & Lin, 2001).

Notation: $\|\cdot\|$ denotes the standard Euclidean norm or the induced matrix 2-norm.

2. System description and preliminaries

In this section, we introduce an adaptive control problem for switched nonlinear systems with constant nonlinearly parameterized uncertainty. We consider the class of switched nonlinear systems of the form

$$\dot{x} = f_{\sigma(t)}(x, u_{\sigma(t)}, \theta_{\sigma(t)}), \tag{1}$$

where $x \in \mathbb{R}^n$ is the system state, for each k, $u_k \in \mathbb{R}^{n_k}$ is the control input of the kth subsystem, $\theta_k \in \mathbb{R}^{q_k}$ is an unknown constant vector. m is the number of subsystems of the switched system (1). The functions $f_k(\cdot)$, $k \in M = \{1, \ldots, m\}$, are assumed to be C^1 with $f_k(0, 0, \theta_k) = 0$. The function $\sigma(t) : [0, +\infty) \to M$ is a switching signal which is assumed to be a piecewise continuous (from the right) function of time. The switching signal $\sigma(t)$ can be characterized by the switching sequence:

$$\Sigma = \left\{x_0;\ (k_0, t_0), (k_1, t_1), \ldots, (k_j, t_j), \ldots | k_j \in M, j \in N\right\}, \tag{2}$$

in which t_0 is the initial time, x_0 is the initial state, N denotes the set of nonnegative integers, and k_j denotes the serial number of the activated subsystem at t_j. When $t \in [t_j, t_{j+1})$, $\sigma(t) = k_j$, that is, the k_jth subsystem is active. The switching sequence Σ is assumed to be minimal in the sense that $k_j \neq k_{j+1}$ for all j. For any $l \in M$, let

$$\Sigma \mid l = \left\{t_{l_1}, t_{l_1+1}, \ldots, t_{l_n}, t_{l_n+1}, \ldots, k_{l_q} = l,\ q \in N\right\},$$

be the sequence of switching times when the lth subsystem is switched on or switched off. In addition, we assume that the state of the system (1) does not jump at the switching instants, i.e., the solution $x(t)$ is everywhere continuous, and there exists at least one k such that $\lim_{j\to\infty}\left(t_{k_j+1} - t_{k_j}\right) \neq 0$.

Our control objective is to solve the problem of adaptive stabilization for the system (1) by design of adaptive controllers for subsystems and a switching law. In this paper, the solvability of the adaptive stabilization problem for subsystems is not assumed. This is because if there exists a subsystem in the system (1) for which the adaptive stabilization problem is solvable, then the adaptive stabilization problem of (1) is trivial.

For the system (1), we need the following assumption.

Assumption 1. σ has finite number of switching on any finite interval of time.

Assumption 1 is a standard assumption in the switching system literature to rule out Zero behavior (Branicky, 1998; Liberzon, 2003).

In the following, we first give a definition and some useful lemmas which will be used in the sequel.

Definition 1 (*Zhao & Hill, 2008*)**.** A function $\alpha : \mathbb{R}_+ \to \mathbb{R}_+$ is called a class $\mathcal{GK}$ function if it is increasing and right continuous at the origin with $\alpha(0) = 0$.

Lemma 1 (*Lin & Qian, 2002a*)**.** *For any real-valued C^0 function $f(x, y)$, where $x \in \mathbb{R}^{q_x}$, $y \in \mathbb{R}^{q_y}$, there exist C^∞ scalar functions $a(x) \geq 0$, $b(y) \geq 0$, $c(x) \geq 1$ and $d(y) \geq 1$, such that $|f(x, y)| \leq a(x) + b(y)$, $|f(x, y)| \leq c(x)d(y)$.*

Lemma 2 (*Lin & Qian, 2002a*)**.** *For $x \in \mathbb{R}$, $y \in \mathbb{R}$, $p \geq 1$ is an integer, the following inequalities hold: $|x+y|^p \leq 2^{p-1}|x^p+y^p|$, $(|x|+|y|)^{1/p} \leq |x|^{1/p} + |y|^{1/p} \leq 2^{(p-1)/p}(|x| + |y|)^{1/p}$. If $p \geq 1$ is an odd integer, then $|x - y|^p \leq 2^{p-1}|x^p - y^p|$.*

Lemma 3 (*Lin & Qian, 2002a*)**.** *Let c, d be positive real numbers and $\alpha(x)$, $\beta(x)$ and $\gamma(x)$ be real-valued continuous functions. For any continuous function $\tau(x) > 0$, there is a continuous function $\rho(x) \geq 0$ such that $|\gamma(x)|\,|\alpha(x)|^c|\beta(x)|^d \leq \tau(x)|\alpha(x)|^{c+d} + |\beta(x)|^{c+d}\rho(x)$.*

Remark 1. By Lemma 1, there exist C^∞ functions $c_k(\theta_k) \geq 1$ and $\gamma_k(x, u_k) \geq 1$ satisfying $\|f_k(x, u_k, \theta_k)\| \leq \gamma_k(x, u_k)c_k(\theta_k)$, $1 \leq k \leq m$. Since θ_k is an unknown constant, $c_k(\theta_k)$ is also an unknown constant. Let $\Theta_k := c_k(\theta_k) \geq 1$ be a new unknown constant. Then, $\|f_k(x, u_k, \theta_k)\| \leq \gamma_k(x, u_k)\Theta_k$, $1 \leq k \leq m$.