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Brief paper

Controllability and stabilizability of a class of systems with higher-order nonholonomic constraints☆

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ABSTRACT

A theoretical framework is established for the control of higher-order nonholonomic systems, defined as systems that satisfy higher-order nonintegrable constraints. A model for such systems is developed in terms of differential–algebraic equations defined on a higher-order tangent bundle. A number of control-theoretic properties such as nonintegrability, controllability, and stabilizability are presented. Higher-order nonholonomic systems are shown to be strongly accessible and, under certain conditions, small time locally controllable at any equilibrium. The applicability of the theoretical development is illustrated through a third-order nonholonomic system example: a planar PPR robot manipulator subject to a jerk constraint. In particular, it is shown that although the system is not asymptotically stabilizable to a given equilibrium configuration using a time-invariant continuous feedback, it is strongly accessible and small-time locally controllable at any equilibrium.

1. Introduction

The problem of controlling dynamical systems that satisfy nonintegrable relations has attracted considerable attention in the recent past. These studies were primarily limited to systems satisfying nonintegrable kinematic relations, also known as systems with first-order (classical) nonholonomic constraints. Examples of systems with nonintegrable first-order constraints include systems subject to rolling constraints as well as mechanical systems that involve symmetries, which result in nonintegrable conserved quantities such as angular momentum. Several examples of systems with first-order nonholonomic constraints have been studied in the context of mobile robots (Samson, 1995; Walsh & Bushnell, 1995; Walsh, Tilbury, Sastry, Murray, & Laumond, 1994) and space robotics (Dubowsky & Papadopoulos, 1993; Nakamura & Mukherjee, 1991). A few representative control works include the study of controllability and stabilizability (Bloch, Reyhanoglu, & McClamroch, 1992), motion planning (Murray & Sastry, 1993; Nakamura & Mukherjee, 1991), and feedback stabilization and tracking (Aneke, 2003; Aneke, Nijmeijer, & de Jager, 2003; Astolfi, 1996; Godhavn & Egeland, 1997; Jiang & Nijmeijer, 1997, 1999; Samson, 1995; Sordalen & Egeland, 1995; Walsh & Bushnell, 1995; Walsh et al., 1994).

In Reyhanoglu, van der Schaft, McClamroch, and Kolmanovsky (1999), the ideas in Bloch et al. (1992) have been extended to dynamical systems that satisfy nonintegrable acceleration relations, i.e., systems with second-order constraints. It has been shown that such systems can arise as models of underactuated mechanical systems, defined as systems with fewer inputs than degrees of freedom (Spong, 1996). Examples of such systems include underactuated vehicles (Aneke et al., 2003; Pettersen & Egeland, 1999; Repoulias & Papadopoulos, 2007) and underactuated manipulators (Aneke, 2003; Baillieul, 1993; Spong, 1995).

Since the beginning of the last century, there has been considerable work on the dynamics formulation of systems with higher-order nonholonomic constraints (Jarzebowska, 2002, 2005, 2006; Ze-chun & Feng-xiang, 1987). Higher-order constraints are defined as program constraints (Jarzebowska, 2002) that arise by imposition of certain design conditions on the allowable motions. As shown in Jarzebowska (2002), the Grioli condition for a rigid body to perform a pseudo-regular precession can be expressed as a second-order nonholonomic program constraint. Program constraints may arise from driving and task requirements in motion planning problems for robotic systems. For example, curvature and

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torsion constraints imposed on robot trajectories can be formulated as second-order and third-order nonholonomic program constraints.

Recently, there has been considerable interest in control of mechanical systems with task and performance conditions involving higher-order time derivatives of the configuration variables (Freeman, 2012; Macfarlane & Croft, 2003; Mellinger & Kumar, 2011). In Macfarlane and Croft (2003), constraints on the permissible jerk are introduced to mitigate the adverse effects of excessive jerk in industrial robot applications. Variety of optimization methods are proposed for generating either minimum-jerk trajectories (Freeman, 2012) or minimum-snap trajectories (Mellinger & Kumar, 2011). These methods are mostly based on parameterization of certain configuration variables via cubic, quartic, or quintic polynomials of time. The motivation of this paper is to develop a theoretical framework for the control of such systems by exploiting the geometric structure as was done in our earlier work in Bloch et al. (1992) and Reyhanoglu et al. (1999). To the best of our knowledge, little has been done in generalizing the control and stabilization ideas developed in Bloch et al. (1992) and Reyhanoglu et al. (1999) to systems with higher-order nonholonomic constraints, except for the work in Jarzebowska (2005, 2006) for the tracking control of such systems.

In this paper, a theoretical framework is presented for the control of higher-order nonholonomic systems defined on a higher-order tangent bundle. A number of control-theoretic properties such as nonintegrability, controllability, and stabilizability are presented. Higher-order nonholonomic systems are shown to be strongly accessible and, under certain conditions, small time locally controllable at any equilibrium. These results are obtained by applying the powerful set of control-theoretic concepts and differential geometric tools developed in the nonlinear control literature (Brockett, 1983; Nijmeijer & van der Schaft, 1990; Sontag et al., 1990; Sussmann, 1979, 1987; Sussmann & Jurdjevic, 1972; Zabczyk, 1989). The novelty of this paper lies in the theoretical results obtained for the class of systems studied in this paper. The applicability of the theoretical framework is illustrated through a physical example. A preliminary version of this paper was presented as a conference paper (Rubio Hervas & Reyhanoglu, 2013).

2. Mathematical model

Consider a system defined on a smooth (C^∞) configuration manifold $\mathbf{Q}$ of dimension n with local coordinates $q = (q^1, \ldots, q^n)$. We introduce higher-order tangent bundles in order to deal with higher-order constraints.

For the configuration manifold $\mathbf{Q}$, the usual tangent bundle is given by Burke (1985) and Crampin and Pirani (1986)

$$T\mathbf{Q} = \bigcup_{q\in\mathbf{Q}} T_q\mathbf{Q},$$

i.e., the union of the tangent spaces to $\mathbf{Q}$. The coordinates q on the configuration manifold $\mathbf{Q}$ generates the natural coordinates $(q, \dot{q})$ for $T\mathbf{Q}$.

The higher-order tangent bundle of the manifold $\mathbf{Q}$ is the fundamental structure of higher-order mechanics. Some basic concepts and notations related to higher-order tangent bundles are now summarized following the development in Gracia, Pons, and Roman-Roy (1991). The *pth order tangent bundle* $T^p\mathbf{Q}$, $p \geq 1$, is defined as an $n(p+1)$-dimensional manifold, whose points are p-velocities. The natural coordinates for $T^p\mathbf{Q}$ are given by $(q, \dot{q}, \ldots, q^{(p)})$. For each l, $0 < l < p$, one can define a projection map $\pi_l^p : T^p\mathbf{Q} \to T^l\mathbf{Q}$ given (in coordinates) by $\pi_l^p(q, \dot{q}, \ldots, q^{(p)}) = (q, \dot{q}, \ldots, q^{(l)})$. The projection map satisfies $\pi_{l'}^l \circ \pi_l^p = \pi_{l'}^p$, $0 \leq l' < l < p$. Here $T^0\mathbf{Q}$ and $q^{(0)}$ are interpreted as $\mathbf{Q}$ and q, respectively. In short, $T^p\mathbf{Q}$ is fibered over $\mathbf{Q}$ as well as $T^l\mathbf{Q}$ for all $0 < l < p$. Note that there is a natural closed embedding $j^p : T^{p+1}\mathbf{Q} \to T(T^p\mathbf{Q})$ defined as $j^p(q, \dot{q}, \ldots, q^{(p+1)}) = (q, \dot{q}, \ldots, q^{(p)}; \dot{q}, \ldots, q^{(p+1)})$.

Systems with first-order (classical) nonholonomic constraints (Bloch et al., 1992) and second-order nonholonomic constraints (Reyhanoglu et al., 1999) naturally appear in several physical examples. In this paper, we extend the developments in Bloch et al. (1992) and Reyhanoglu et al. (1999) to systems with *higher-order nonholonomic constraints* that are affine in the highest-order derivatives, i.e., constraints of the form

$$A(q, \dot{q}, \ldots, q^{(p-1)})q^{(p)} + S(q, \dot{q}, \ldots, q^{(p-1)}) = 0, \quad p \geq 1, \tag{1}$$

where $A \in \mathbb{R}^{r\times n}$ and $S \in \mathbb{R}^r$, $1 \leq r < n$, are smooth (C^∞) functions defined on appropriate subsets of $T^{p-1}\mathbf{Q}$.

Assume that there exists a non-constant smooth function

$$h(\cdot) : \mathbb{R} \times T^{p-1}\mathbf{Q} \to \mathbb{R}$$

such that $\frac{dh}{dt} = 0$ along the solutions of dynamic equations, then h is called a *non-trivial first integral.*

Definition 1. The constraints (1) are said to be (completely) pth order nonholonomic if there does not exist any non-trivial first integral.

Remark 1. The sth order jet prolongation of $\mathbb{R} \times \mathbf{Q}$ is denoted by $\mathcal{J}^s(\mathbb{R} \times \mathbf{Q})$. If (t, q) are fibered coordinates on $\mathbb{R} \times \mathbf{Q}$, then $(t, q, \dot{q}, \ldots, q^{(s)})$ are fibered coordinates on $\mathcal{J}^s(\mathbb{R}\times\mathbf{Q})$. Clearly, one can identify $\mathcal{J}^s(\mathbb{R}\times\mathbf{Q})$ with $\mathbb{R}\times T^s\mathbf{Q}$. Thus $h(t, q, \dot{q}, \ldots, q^{(p-1)})$ can also be viewed as a scalar function defined on the (p-1)th order jet prolongation.

Assume that the $r \times n$ matrix A in Eq. (1) has full rank. Then there is no loss of generality in assuming that the generalized coordinates are ordered so that the last r columns of the matrix A constitute an $r \times r$ locally invertible matrix function, i.e., A can be expressed as $[A_1 \quad A_2]$, where A_1 is an $r \times m$, $m = n - r$, matrix function and A_2 is an $r \times r$ locally nonsingular matrix function. Let $q = (q_1, q_2)$, $q_1 \in \mathbb{R}^m$, $q_2 \in \mathbb{R}^r$ be a partition of generalized coordinates corresponding to the partitioning of the matrix function A. Eq. (1) can now be rewritten as

$${q_2}^{(p)} = J(q, \dot{q}, \ldots, q^{(p-1)}){q_1}^{(p)} + R(q, \dot{q}, \ldots, q^{(p-1)}), \tag{2}$$

where $J = -A_2^{-1}A_1 \in \mathbb{R}^{r\times m}$ and $R = -A_2^{-1}S \in \mathbb{R}^r$ are assumed to be smooth functions defined on appropriate subsets of $T^{p-1}\mathbf{Q}$.

Second-order relations of the above form appear naturally in systems under the action of $m < n$ independent control forces and/or torques; i.e., systems with fewer control inputs than degrees of freedom (see e.g. Reyhanoglu et al., 1999 and references therein). For such systems q_1 and q_2 represent the actuated and unactuated degrees of freedom, respectively. Examples of systems with second-order constraints include underactuated vehicles (Aneke et al., 2003; Pettersen & Egeland, 1999; Repoulias & Papadopoulos, 2007) and underactuated manipulators (Aneke, 2003; Baillieul, 1993; Spong, 1995).

3. Controllability and stabilizability results

In this section, we study controllability and stabilizability properties of higher-order nonholonomic systems. A powerful set of tools for the analysis of the properties of such systems is based on concepts derived from differential geometry. The reader is referred to Brockett (1983); Nijmeijer and van der Schaft (1990); Sontag et al. (1990); Sussmann (1979, 1987); Sussmann and Jurdjevic (1972); Zabczyk (1989) for the controllability and stabilizability concepts developed in the nonlinear control literature.

Let $(q, \dot{q}, \ldots, q^{(p-1)})$ for $q \in \mathbb{R}^n$ denote local coordinates on the $(p-1)$th order tangent bundle $\mathbf{M} = T^{p-1}\mathbf{Q}$, where p refers to the order of the nonholonomic constraint. Generalizing the ideas introduced in Reyhanoglu et al. (1999), we define the r-covector