Automatica 54 (2015) 256–265

Contents lists available at ScienceDirect

Automatica

journal homepage: www.elsevier.com/locate/automatica

Brief paper

Adaptive backstepping control of uncertain linear systems under unknown actuator delay[☆]

Yang Zhu [a,b], Hongye Su [a,1], Miroslav Krstic [b]

[a] State Key Laboratory of Industrial Control Technology, Institute of Cyber-Systems and Control, Zhejiang University, Hangzhou, 310027, PR China

[b] Department of Mechanical and Aerospace Engineering, University of California, San Diego, La Jolla, CA 92093-0411, USA

ARTICLE INFO

Article history:
Received 27 June 2014
Received in revised form
7 November 2014
Accepted 23 January 2015
Available online 24 February 2015

Keywords:
Adaptive
Backstepping
Output feedback
Prediction-based
Boundary
Time-delay

ABSTRACT

Trajectory tracking of uncertain time-delay linear systems by output feedback control is of theoretical importance and practical value. In this paper, concentrating on a class of linear plants whose relative degree equals to system dimension, we develop a Lyapunov-based control scheme to achieve trajectory tracking despite some classic difficulties including unmeasurable system state, unknown plant parameters and unknown input time-delay. A comprehensive approach combining adaptive backstepping output feedback with prediction-based boundary control is employed in the design. The stability analysis exhibits the global boundedness of all closed-loop system signals and the tracking performance is also guaranteed.

1. Introduction

Actuator time-delay exists widely in practice such as process control of chemical engineering, industrial oil-drilling plants, temperature control of air-conditioning and so on. Some of them may arise in an actual physical transport delay, while others may demonstrate themselves as computational delays. As a consequence, an increasing number of research have focused on the control of unstable plants with a long actuator delay over past four decades. Based on the notation of a predictor feedback (Artstein, 1982; Evesque, Annaswamy, Niculescu, & Dowling, 2003; Gu & Niculescu, 2003; Kwon & Pearson, 1980; Manitius & Olbrot, 1979; Michiels & Niculescu, 2003; Mirkin, 2004; Mondie & Michiels, 2003; Niculescu & Annaswamy, 2003; Richard, 2003; Watanabe & Ito, 1981; Zhong, 2006), the input delay obtained a perfect compensation and the systems achieved the stabilization. Nevertheless, most of these results required that the delay value was known. Adaptive control of time-delay systems successfully dealt with uncertain parameters in the system (Evesque et al., 2003; Niculescu & Annaswamy, 2003; Ortega & Lozano, 1998), however, they also called for a known delay value for design. Zhou, Wen, and Wang (2009) studied the robustness of adaptive backstepping control for linear systems to dynamic perturbations including input delay, but the proposed feedback scheme did not aim at compensating for the delay effect. The significance of control problems with unknown delay was emphasized in Diop, Kolmanovsky, Moraal, and Van Nieuwstadt (2001) and Krstic and Banaszuk (2006), in which an approximation design was applied to a limited class of plants.

Later on, a new kind of prediction-based boundary control (Krstic & Smyshlyaev, 2008), has been developed to address the delay uncertainty, in which the original system is considered as the Ordinary Differential Equation (ODE) part, while the actuator delay is regarded as a transport Partial Differential Equation (PDE) part and the ODE–PDE cascade system has been studied as a whole. As stated in Chapter 7.1 of Krstic (2009), there exist a total number of 14 distinct problem combinations due to unknown input delay, unknown plant parameters, unmeasured ODE state and unmeasured PDE state. A few of these problems like a composition of unknown delay and parameters, a composition of unmeasurable ODE and PDE states has been addressed very well in Bresch-Pietri, Chauvin, and Petit (2012), Bresch-Pietri and Krstic (2009) and Bresch-Pietri and Krstic (2010).

☆ This work received support from China Scholarship Council (CSC) and National Natural Science Foundation of China (NSFC: 61134007, 61320106009, 61304012). The material in this paper was not presented at any conference. This paper was recommended for publication in revised form by Associate Editor Hiroshi Ito under the direction of Editor Andrew R. Teel.

E-mail addresses: zhuyang@iipc.zju.edu.cn, zhy024@ucsd.edu (Y. Zhu), hysu@iipc.zju.edu.cn (H. Su), krstic@ucsd.edu (M. Krstic).

[1] Tel.: +86 571 87951075; fax: +86 571 87952279.

http://dx.doi.org/10.1016/j.automatica.2015.02.013
0005-1098/

In this paper, we propose a control scheme to achieve trajectory tracking despite unmeasurable ODE state, unknown system parameters and unknown input time-delay, which is one of 14 problems and has never been considered yet. The systems that we focus on are a class of linear plants whose relative degree is equal to system dimension. The method that we perform here is a comprehensive approach combining adaptive backstepping output feedback control in Krstic, Kanellakopoulos, and Kokotovic (1995) and Zhou et al. (2009) with prediction-based boundary control in Bresch-Pietri and Krstic (2009) and Krstic (2009). It seems appropriate to say this is not a trivial composition of existing results while, as shown in later derivation, a few of relatively new technical issues grow up in dealing with this problem. First of all, by modeling the system as an observer canonical form, the somewhat restrictive assumptions such that a completely controllable pair $(A(\theta), B(\theta))$ in Bresch-Pietri et al. (2012), Bresch-Pietri, Chauvin, and Petit (2014) and Bresch-Pietri and Krstic (2009) have been removed. In addition, a class of Kreisselmeier-filters (K-filters) are brought into virtually estimate unmeasurable ODE state which makes controller design much more sophisticated than Bresch-Pietri and Krstic (2009). More importantly, to achieve a linear controller to apply the prediction control, we do not employ tuning functions or nonlinear damping terms like Krstic et al. (1995) and Zhou et al. (2009) to address parameter estimation error on the backstepping Lyapunov-based design. Instead, we appeal to adaptation gains (both for parameter and delay) and normalization of update laws to make ODE–PDE cascade system stable. In terms of trajectory tracking, the reference output filter and reference input filter are introduced to help get a backstepping transformation for boundary control. In fact, there is no result about Lyapunov-based adaptive backstepping control without overparametrization, tuning functions and nonlinear damping. We have to acknowledge that we do not manage to extend our design to a more general case—when the relative degree is less than the system dimension. In that situation, the remaining zero dynamics of the plant should be incorporated in the design to globally stabilize the closed-loop system, however, this is impossible because of unmeasurable ODE state and unknown parameters.

The rest of the paper is organized as follows. In Section 2, we present the plant model and formulate the control problem and objectives. In Section 3, the Kreisselmeier-filters are used to estimate the unmeasured ODE state. The controller and identifier are presented in Sections 4 and 5, respectively. The stability for the whole closed-loop system and the trajectory tracking are analyzed in Section 6. An numerical example is illustrated in Section 7 followed by the conclusion of the paper in Section 8.

2. System description and problem formulation

We consider a class of linear single-input single-output systems which can be represented as the following observer canonical form:

$$\dot{X}(t) = AX(t) - aY(t) + bU(t-D)$$
$$Y(t) = X_1(t) \tag{1}$$

where

$$A = \begin{bmatrix} 0 & & \\ \vdots & I_{n-1} & \\ 0 & \cdots & 0 \end{bmatrix}, \qquad a = \begin{bmatrix} a_{n-1} \\ \vdots \\ a_1 \\ a_0 \end{bmatrix}, \qquad b = \begin{bmatrix} 0 \\ \vdots \\ 0 \\ b_0 \end{bmatrix} \tag{2}$$

and $X(t) = [X_1(t), X_2(t), \ldots, X_n(t)]^T \in \mathbb{R}^n$ is the unmeasured state vector, $Y(t) \in \mathbb{R}$ is the measurable output, $U(t-D) \in \mathbb{R}$ is the input with an unknown constant time-delay D, while $a_{n-1}, \ldots, a_1, a_0$ and b_0 are unknown constant plant parameters and control coefficient, respectively.

Remark 1. Comparing (1) with Eq. (10.3) in Chapter 10 of Krstic et al. (1995), one has no trouble finding that (1) is a special case of (10.3) with $b_m = b_{m-1} = \cdots = b_1 = 0$, which means the relative degree is identical to the system dimension. We have to admit that this kind of system is restrictive to some extent, but to the best of our knowledge, there are still many linear delayed plants in practice satisfying this kind of structure, please refer to examples and simulations in Bresch-Pietri et al. (2012) and Bresch-Pietri et al. (2014). When the relative degree is less than the system dimension, to achieve the stabilization of the closed-loop system, one can see that the remaining m-dimensional zero dynamics ζ just for analysis in (10.133) of Krstic et al. (1995) should be included in the design of the normalized update law (75) of this paper. However, this is obviously impossible since ζ is unmeasured and $b_m, b_{m-1}, \ldots, b_0$ are unknown.

For convenience of description, we rewrite plant (1) compactly as

$$\dot{X}(t) = AX(t) + F\big(U(t-D), Y(t)\big)^T\theta$$
$$Y(t) = e_1^T X(t) \tag{3}$$

where $p = 1 + n$ dimensional parameter vector θ is defined by $\theta = [b_0, a_{n-1}, \ldots, a_1, a_0]^T$ and e_i for $i = 1, 2, \ldots$ is the ith coordinate vector in corresponding space,

$$F\big(U(t-D), Y(t)\big)^T = \left[\begin{bmatrix} 0_{(n-1)\times 1} \\ 1 \end{bmatrix} U(t-D), \quad -I_n Y(t)\right]. \tag{4}$$

Our control objectives are listed as follows:

- Design a control scheme to compensate for the system uncertainty and actuator time-delay to ensure that all the signals of the closed-loop system are globally bounded.
- Make output $Y(t)$ asymptotically track a reference signal $Y_r(t)$.

To achieve the above control objectives, we have the following assumptions.

Assumption 1. In the case of known θ, given a time-varying reference output trajectory $Y_r(t)$ which is known, bounded and smooth, there exist known reference state signal $X^r(t, \theta)$ and reference input signal $U^r(t, \theta)$ which are bounded in t, continuously differentiable in the argument θ and satisfy

$$\dot{X}^r(t, \theta) = AX^r(t, \theta) + F\big(U^r(t, \theta), Y_r(t)\big)^T\theta$$
$$Y_r(t) = e_1^T X^r(t, \theta). \tag{5}$$

Remark 2. Assumption 1 is a mild variation of traditional Assumption 10.4 on p. 418 of Krstic et al. (1995). Suppose $Y_r(t)$ and its first n derivatives are known, bounded and piecewise continuous. If we choose reference states as $X_1^r(t) = Y_r(t), X_2^r(t, \theta) = \dot{Y}_r(t) + a_{n-1}Y_r(t), \ldots, X_n^r(t, \theta) = Y_r^{(n-1)}(t) + a_{n-1}Y_r^{(n-2)}(t) + a_{n-2}Y_r^{(n-3)}(t) + \cdots + a_2\dot{Y}_r(t) + a_1 Y_r(t)$, it is easy to find a known, bounded $U^r(t, \theta)$ to make the equality $Y_r^{(n)}(t) + a_{n-1}Y_r^{(n-1)}(t) + a_{n-2}Y_r^{(n-2)}(t) + \cdots + a_1\dot{Y}_r(t) + a_0 Y_r(t) = b_0 U^r(t, \theta)$ hold, thus (5) is checked.

Assumption 2. There exist two known finite constants $\underline{D} > 0$ and $\bar{D} > 0$, such that $D \in [\underline{D}, \bar{D}]$. The sign of the high-frequency gain b_0, i.e. $\mathrm{sgn}(b_0)$, is known and there exist two known finite constants $\underline{b}_0$, $\bar{b}_0$ such that $0 < \underline{b}_0 \le |b_0| \le \bar{b}_0$. In addition, there exists a convex compact set $\mathcal{A} \subset \mathbb{R}^n$ such that $\exists \bar{a}, a^*, |a - a^*| \le \bar{a}$ for all $a \in \mathcal{A}$, where $a^* \in \mathbb{R}^n$ is a known constant vector, $\bar{a} > 0$ is a known finite constant.