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Brief paper

Model predictive control for constrained networked systems subject to data losses ☆

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ABSTRACT

The paper addresses the stabilization problem for constrained control systems where both plant measurements and command signals in the loop are sent through communication channels subject to time-varying delays and data losses. A novel receding horizon strategy is proposed by resorting to an uncertain polytopic linear plant framework. Sequences of pre-computed inner approximations of the one-step controllable sets are on-line exploited as target sets for selecting the commands to be applied to the plant in a receding horizon fashion. The communication channel effects are taken into account by resorting to both Independent-of-Delay and Delay-Dependent stability concepts that are used to initialize the one-step controllable sequences. The resulting framework guarantees Uniformly Ultimate Boundedness and constraints fulfilment of the regulated trajectory regardless of plant uncertainties and data loss occurrences.

1. Introduction

Networked Control Systems (NCS) represent the interconnection of a set of plants equipped with sensing and communication devices. From an abstract point of view an NCS can be regarded as a system comprised of the plant to be regulated and of actuators, sensors, and controllers, coordinated via a communication channel. NCS stability analysis and control design features are attracting considerable attention in the technical literature, see Franzè, Famularo, and Tedesco (2011), Hespanha, Naghshtabrizi, and Xu (2007), Montestruque and Antsaklis (2004) and references therein. Recent contributions and tutorials on NCS modeling and performance analysis have been conducted using discrete-time (Cloosterman, van de Wouw, Heemels, & Nijmeijer, 2009), sampled data (Fridman & Shaked, 2005) and continuous time (Heemels, Teel, van de Wouw, & Nesic, 2010) framework approaches, respectively.

Of interest here are constrained Receding Horizon Control strategies which are an extremely appealing methodology for NCS stabilization due to their intrinsic capability to generate, at each time instant, a sequence of virtual inputs which can be transmitted within a single data-packet (Quevedo, Silva, & Goodwin, 2007). Noticeable contributions on this matter are from Muñoz de la Peña and Christofides (2008), Pin and Parisini (2011) and Quevedo and Gupta (2013): in Muñoz de la Peña and Christofides (2008), the authors consider a receding horizon strategy for nonlinear networked systems under wireless and asynchronous measurement sampling. Quevedo and Gupta (2013) extend a control scheme for nonlinear plants, popular in real-time systems, to tolerate the presence of time-varying processing resources (such as variable delays, packet losses/drops etc.), known as *anytime* algorithm. In Pin and Parisini (2011), following the same ideas as Muñoz de la Peña and Christofides (2008), a nonlinear RHC scheme exploiting a Network Delay Compensation strategy is proposed to efficiently manage the simultaneous presence of constraints, model uncertainties, time-varying transmission delays and data-packet losses.

We will focus on a novel discrete time receding horizon strategy for NCSs, described by means of uncertain multi-model linear systems, under the occurrence of time-varying delays, data loss on the sensor-to-controller link and feedback command loss on the controller-to-actuator link. By resorting to a time-stamp protocol, data and feedback losses are separately accounted to make available a "usable" control move for the actuator logic within each sampling interval. The NCS stabilization problem will be dealt with a dual-mode predictive strategy. Off-line families of one-step controllable sets, "capable" to efficiently manage normal and data loss phases on actuator–sensor sides are first obtained by resorting to Independent-of-Delay (**IOD**) and Delay-Dependent (**DD**) stability concepts. Then at each sample time, an on-line receding horizon

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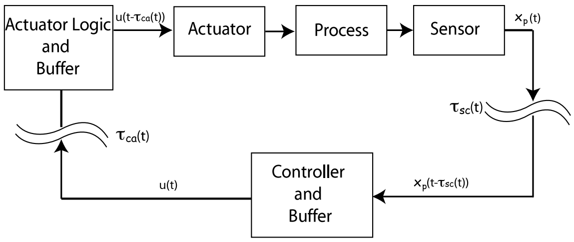

Fig. 1. Networked control system structure.

scheme is computed by deriving the smallest ellipsoidal set (**DD** or **IOD** type) compliant with the delay scenario.

The main merits of the proposed strategy can be summarized as follows: (1) the computation of one-step controllable ellipsoids sequences capable to cope with time-delays occurrences; (2) the computational resources (CPU power, memory resources and bandwidth requirements) are significantly modest when contrasted with competitor schemes.

2. Problem formulation

We will refer the reader to the networked control scheme depicted in Fig. 1 where delay effects are taken into consideration from the sensor and actuator sides. Specifically:

Process—It is described by a multi-model uncertain discrete-time linear system

$$x_p(t+1) = \Phi(\alpha(t))x_p(t) + G(\alpha(t))u(t) \tag{1}$$

where $t \in \mathbb{Z}_+ := \{0, 1, \ldots\}$, $x_p(t) \in \mathbb{R}^n$ denotes the state plant and $u(t) \in \mathbb{R}^m$ the control input. The parameter vector $\alpha(t) \in \mathbb{R}^l$ is assumed to lie in the unit simplex

$$\mathcal{P}_l := \left\{\alpha \in \mathbb{R}^l : \sum_{i=1}^{l} \alpha_i = 1,\ \alpha_i \geq 0\right\} \tag{2}$$

and the system matrices $\Phi(\alpha)$ and $G(\alpha)$ belong to

$$\Sigma(\mathcal{P}_l) := \left\{(\Phi(\alpha), G(\alpha)) = \sum_{i=1}^{l} \alpha_i(\Phi_i, G_i), \alpha \in \mathcal{P}_l\right\} \tag{3}$$

the pairs (Φ_i, G_i) representing the polytope vertices $\Sigma(\mathcal{P}_l)$, viz. $(\Phi_i, G_i) \in \text{vert}\{\Sigma(\mathcal{P}_l)\}$, $\forall i \in \underline{l} := \{1, 2, \ldots, l\}$. Moreover, the control input is subject to the following saturation constraints:

$$u(t) \in \mathcal{U},\ \forall t \geq 0, \quad \mathcal{U} := \{u \in \mathbb{R}^m \,|\, u^T u \leq \bar{u}\}, \tag{4}$$

with $\bar{u} > 0$ and $\mathcal{U}$ a compact subset of $\mathbb{R}^m$ containing the origin as an interior point.

Actuator and controller buffers—The actuator buffer is in charge to memorize the last received command, hereafter denoted as u_{-1}^R, whereas the buffering unit on the controller side stores the last measurement received from the sensor-to-controller channel, named x_{-1}, and the last computed command u_{-1}^C.

Actuator logic—The actuator tracks data losses on the feedback channel: at each time instant t such a logic is instructed to apply the command $u(t)$ if available or conversely u_{-1}^R.

To properly treat data loss both on the plant–controller and controller–plant links, the sensor-to-controller and the controller-to-actuator cases need to be separately analyzed. We will suppose first that the delay on the command channel side $\tau_{ca}(t) : \mathbb{Z}_+ \to \mathbb{Z}_+$ is such that $\tau_{ca}(t) \leq \bar{\tau}_c$, $\forall t$, while the delay on the measurement side $\tau_{sc}(t) : \mathbb{Z}_+ \to \mathbb{Z}_+$ could be unbounded. On the contrary, the controller-to-actuator link (the feedback) is only subject to the actual induced delay $\tau_{ca}(t)$. Moreover, at each time instant t we shall define respectively with $\tau_m(t) \leq \tau_{ca}(t)$ and $\tau_c(t) \leq \tau_{sc}(t)$ the *age* of the state measurement used by the controller to compute the input and the *age* of the command used by the actuator. Then, on the plant–controller link at each time instant t when computing the input $u(t)$, the following age cumulative network latency should in principle be used: $\tau_{NL}(t) = \tau_m(t) + \tau_c(t - \tau_m(t))$. Since it is well-known that the round-trip delay $\tau_c(t-\tau_m(t))$ cannot be available at the controller side, the upper bound $\bar{\tau}_c$ on the controller–actuator link during both the controller design and the command computation $u(\cdot)$ phases is considered, i.e.

$$\tau(t) = \tau_m(t) + \bar{\tau}_c, \quad \forall t. \tag{5}$$

Hence, the following time-delay scenarios can occur on the communication channels:

- Sensor-to-Controller link
  - **Normal phase**—each time-delay occurrence is bounded, $\tau(t) < \bar{\tau}$, $\bar{\tau}$ being the maximum allowable transmission interval (MATI) (Walsh, Beldiman, & Bushnell, 2001);
  - **Data loss**—there exists $\bar{t}$ such that $\tau(\bar{t}) \geq \bar{\tau}$; the state measurement will be no longer available for the control action computation.
- Controller-to-Actuator link
  - **Normal phase**—at each time instant t the actuator receives a control action $u(t)$;
  - **Feedback loss**—there exists $\bar{t}$ such that $\tau_c(\bar{t}) \geq 1$, no control action is available for feedback.

Then, the problem statement is:

Network Constrained Stabilization (NCS) problem—Given the networked system depicted in Fig. 1 and the model plant (1)–(3), determine a state-feedback regulation strategy

$$u(\cdot) = g(x_p(\cdot)) \tag{6}$$

complying with (4) such that, in the presence of time-delays (**Normal phases**) and packet dropouts (**Data losses**, **Feedback losses**) on both the communication channels, the regulated state trajectory is "jailed" inside the domain of attraction (DoA) due to (6) (Uniformly Ultimate Boundedness). □

In what follows we will be inspired by the class of computationally low demanding MPC schemes proposed e.g. in Angeli, Casavola, Franzè, and Mosca (2008) and Wan and Kothare (2003) which will be properly adapted to the NCS framework of Fig. 1. To this end, the following definition will be used (Blanchini & Miani, 2008; Raković, Kerrigan, Mayne, & Lygeros, 2006):

Definition 1. Given the plant (1) and a controlled-invariant target set $\mathcal{T}$, the set of states i-step controllable to $\mathcal{T}$ is defined via the following recursion:

$$\begin{aligned} &\mathcal{T}_0 := \mathcal{T} \\ &\mathcal{T}_i := \{x_p : \exists u \in \mathcal{U} : \Phi(\alpha)x_p + G(\alpha)u \in \mathcal{T}_{i-1},\ \forall \alpha \in \mathcal{P}_l\} \end{aligned} \tag{7}$$

where $\mathcal{T}_i$ is the set of states that can be steered into $\mathcal{T}_{i-1}$ using a single control move.

Then, the structure of the proposed RHC algorithm is:

- *Off-line*—Two stabilizing state-feedback control laws (6) and the corresponding robust positively invariant regions (RPI) $\mathcal{T}_0^{DD}$ and $\mathcal{T}_0^{IOD}$ for (1)–(3) are first derived by resorting to **DD** and **IOD** stability concepts, respectively. Then, two sequences of N one-step ahead controllable sets $\{\mathcal{T}_i^{DD}\}$ and $\{\mathcal{T}_i^{IOD}\}$ are computed by enlarging $\mathcal{T}_0^{DD}$ and $\mathcal{T}_0^{IOD}$ under the requirement that each new state can be steered into $\mathcal{T}_0^{DD}$ (respectively $\mathcal{T}_0^{IOD}$) in a finite number of steps.