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Brief paper

Stochastic games of resource extraction[☆]

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ABSTRACT

We study stochastic games of resource extraction in which the transition probability is a convex combination of stochastic kernels with coefficients depending on the joint investments of the players. Our approach covers the unbounded utility case which was not examined in this class of games beforehand. We give two theorems on the existence of pure stationary Markov perfect equilibria for the models of games under consideration. A detailed discussion with illustrative examples explains the meaning of our assumptions and their relation to the conditions used earlier in the literature.

1. Introduction

The role of various models of resource extraction dynamic games in economics is extensively discussed in a survey article by Van Long (2011). In this paper, we study a strategic version of the well-known discrete-time one-sector optimal growth model (see Bhattacharya & Majumdar, 2007 or Stokey, Lucas, & Prescott, 1989), which plays a principal role in economic dynamics, in particular, in macroeconomics and in resource economics. In principle, it can be described as follows. There are two agents who jointly own natural resource and who consume some amount of the available stock at each stage in order to maximise their individual discounted sum of utilities. The transition law of the next state evolves according to the equation $s_{t+1} = p(s_t - a_t - b_t)$ $(t = 0, 1, \ldots)$, where s_t is the stock level, a_t and b_t are the consumption levels of players 1 and 2 at time t and $p(\cdot)$ is a natural growth law or production function mapping investments into the next stock. Without any extraction capacities this is a generalised game, in which the two simultaneous actions at time t are jointly constrained in a natural way by $a_t + b_t \le s_t$, $a_t \ge 0$, $b_t \ge 0$. The seminal model of Levhari and Mirman (1980) is concerned with a specific version of the aforementioned model with the logarithmic one-period utility functions for both players and production function $p(s_t) = s_t^{\alpha}$, $\alpha \in (0, 1)$.

The model of Levhari and Mirman (1980) has been studied in more complex resource games. For instance, Sundaram (1989) proved under the symmetry assumption on the payoff structure that the game admits a stationary Markov perfect equilibrium in non-randomised Markov strategies. His result was later extended to stochastic games of resource extraction by Dutta and Sundaram (1992) and Majumdar and Sundaram (1988). It is worthy to emphasise that Majumdar and Sundaram (1988) considered only atomless transitions, whereas Dutta and Sundaram (1992) dealt with a pretty general stochastic model that included deterministic transitions as a special case. However, there is one serious limitation of the approaches used in the aforementioned works. Namely, Dutta and Sundaram (1992) and Majumdar and Sundaram (1988) assume that the players have the same utility functions or, in other words, the identical preferences. Such games are called *symmetric*. Without this symmetry condition the problem of proving existence of a Nash equilibrium in a general stochastic game of resource extraction becomes very difficult, because the state space is uncountable and the parts of discounted utilities involving expectations with respect to the transitions probabilities are not concave in actions. The methods used by Dutta and Sundaram (1992), Majumdar and Sundaram (1988) or Sundaram (1989) in the symmetric case do not work in a general set-up, when the symmetry assumption is dropped.

The stochastic games of resource extraction without the symmetry condition were examined by Amir (1996a), who considered so-called "convex transitions". More precisely, he assumed that the conditional cumulative distribution function induced by the transition probability is convex with respect to investments. Using certain ideas from lattice programming (see Topkis, 1978) he proved

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the existence of pure stationary Markov perfect equilibria in the class of Lipschitz continuous strategies. His assertions are pretty strong (e.g., Lipschitz continuity of equilibria, monotonicity of the equilibrium payoffs in the state variable), but the convexity assumption imposed on the transition functions is very restrictive. To the best of our knowledge, there are only two examples in the literature that satisfy Amir's convexity conditions and they hold rather uncommon properties. One example concerns the class of transitions that are convex combinations of some probability measures on the state space with coefficients depending on investments. This class was also considered in Nowak (2003) or in Balbus and Nowak (2008). However, as we argue in Section 3, this type of transition probabilities makes sense only in the bounded state space case. A careful analysis of various examples suggests that the convexity assumption made by Amir (1996a,b) is, in fact, satisfied very rarely. Usually, the cumulative distribution induced by the transition probability is neither convex nor concave with respect to investments. A further discussion is provided in Remarks 7 and 8 in Section 3, where we also show that from an application point of view transition functions being a convex combination of stochastic kernels depending on the state variable are more desirable. The coefficients of the combination, as in the previous works, can depend on joint investments of the players. This type of transitions was introduced in Balbus and Nowak (2004) to study symmetric games of resource extraction and discussed in examples given by Horst (2005), but in a different context of dynamic stochastic production games. The assumptions made in this paper enable us to prove the existence of pure stationary Markov perfect equilibria in a certain class of stochastic games of resource extraction with (typical in the theory of economic growth) unbounded concave utility functions for the players. All previous considerations have been devoted only to the bounded utility case. The games that we deal with do not usually belong to the class of supermodular stochastic games in the sense of Curtat (1996). Unlike Amir (1996a), we are not able to apply the techniques of lattice programming given by Curtat (1996), Milgrom and Roberts (1990) and Topkis (1978). Moreover, the comparative statics or monotonicity results provided by Milgrom and Roberts (1990) for static games are not useful within our framework, since the transition functions can be neither convex in the sense of Amir (1996a) nor supermodular in the sense of Curtat (1996). On the other hand, our assumption on the transition probability function seems to be more natural in some models. The descriptions of two game models with the proofs of the existence of pure stationary Markov equilibria are given in Section 2. Section 3 contains a detailed discussion of our assumptions and earlier works in this area.

2. The models and main results

2.1. Model I

Let $\mathbb{N}$ be the set of positive integers and $\mathbb{R}$ be the set of all real numbers. We consider an m-person discounted stochastic game for which:

(i) $S = (0, M)$ with $0 < M \le \infty$ is the *state space*.

(ii) $A_k(s) = [0, b_k(s)]$ is the *set of actions* available to player k in state $s \in S$. For each k, the function $b_k : S \mapsto S$ is continuous and increasing. Moreover, $\sum_{k=1}^m b_k(s) < s$ for all $s \in S$. For any $s \in S$, define

$$A(s) = A_1(s) \times \cdots \times A_m(s)$$

and

$$D := \{(s, a) : s \in S,\ a \in A(s)\}.$$

(iii) $u_k : [0, M) \mapsto \mathbb{R}$ is a non-negative increasing twice differentiable utility function u_k for player k is such that $u_k(0) = 0$. We assume that $u_k'' < 0$.

(iv) For any $s \in S$ and $a \in A(s)$, $p(\cdot|s, a)$ is a probability measure on S. For $a = (a_1, \ldots, a_m) \in A(s)$, we define $\sigma(a) := \sum_{k=1}^m a_k$ and assume that there are transition probabilities q_1 and q_2 from S to S such that

$$p(\cdot|s, a) = \frac{\sigma(a)}{s} q_1(\cdot|s) + \frac{s - \sigma(a)}{s} q_2(\cdot|s). \tag{1}$$

Moreover, each q_i is dominated by a σ-finite measure μ defined on S, i.e., for any Borel set $B \subset S$ and $i = 1, 2$, we have

$$q_i(B|s) = \int_B \rho_i(s, s')\mu(ds'),$$

where $\rho_i(s, \cdot)$ is the Radon–Nikodym derivative for $q_i(\cdot|s)$ with respect to μ. In addition, there exists a continuous function $w : [0, M) \mapsto [1, +\infty)$ such that $u_k(s) \le w(s)$ for all $s \in [0, M)$, $k \in \{1, \ldots, m\}$ and there exists a constant α such that

$$\int_S w(s')q_i(ds'|s) \le \alpha w(s) \quad \text{for } i = 1, 2 \tag{2}$$

and $\alpha\beta < 1$, $\beta \in (0, 1)$.

(v) β is a *discount coefficient*.

(vi) For $i = 1, 2$, we suppose that if $s_n \to s_0$, then

$$\int_S |\rho_i(s_n, s') - \rho_i(s_0, s')| w(s')\mu(ds') \to 0. \tag{3}$$

The above components are used to define a discrete-time dynamic game in which the economic interaction may be described as follows: m players jointly own a single good or productive asset characterised by a stochastic production function. In each period they observe a state $s \in S$ and simultaneously choose their actions $(a_1, \ldots, a_m) \in A(s)$ that yield the utility vector $u(a) = (u_1(a_1), \ldots, u_m(a_m))$. A new state s' is realised from the distribution $p(\cdot|s, a)$ and new period begins with utilities discounted by β.

Observe that by (ii) the stock cannot be driven to 0 by exhaustive consumption by the players. Hence, the game will effectively last forever. Such an assumption of non-exhaustive capacities has previously been made in the literature, see the comment on p. 117 in Amir (1996a). Here we would like to mention that our model can be extended to the state space $S = [0, M)$. Then, we may allow to have that $\sum_{k=1}^m b_k(s) \le s$. The transition probability can be defined as follows: it is same as in (1) for $s \in (0, M)$ and $p(\cdot|0, (0, \ldots, 0)) = \alpha_0\delta_0(\cdot) + (1 - \alpha_0)\mu(\cdot)$ for some $\alpha_0 \in (0, 1)$ and a probability measure μ on $(0, M)$. By δ_0 we denote the Dirac measure concentrated at 0. In this context, there is a positive probability of leaving state 0 (see footnote 27 in Balbus, Reffett, & Woźny, 2013b, where such a model is commented).

A *strategy* for a player is a sequence of Borel measurable mappings from the history space to the space of actions available to her/him. The set of strategies for player k is denoted by Π_k and its generic element by π_k. Let F_k be the set of all Borel measurable functions $f_k : S \mapsto [0, M)$ such that $f_k(s) \in A_k(s)$ for each $s \in S$. A *stationary Markov strategy* for player k is a constant sequence $(\pi_{kt})_{t\in\mathbb{N}}$ where $\pi_{kt} = f_k$ for some $f_k \in F_k$ and for all $t \in \mathbb{N}$. Hence, a stationary Markov strategy for player k can be identified with the Borel measurable mapping $f_k \in F_k$. We put $F = F_1 \times \cdots \times F_m$. For any $\pi = (\pi_1, \ldots, \pi_m) \in \Pi_1 \times \cdots \times \Pi_m$, an initial state $s \in S$ and $t \in \mathbb{N}$, by $u_k^{(t)}(\pi)(s)$ we denote the *expected utility* for player k in the t-th period of the game. The *expected discounted* utility for player k is

$$U_k(s, \pi) = \sum_{t=1}^{\infty} \beta^{t-1} u_k^{(t)}(\pi)(s).$$