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Brief paper

Model predictive control for constrained systems with serially correlated stochastic parameters and portfolio optimization ☆

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ABSTRACT

In this paper, we consider MPC for constrained discrete-time systems with stochastic parameters which are assumed to be a set of serially correlated time series. A generalized performance criterion is composed of a weighted sum of a linear combination of the (a) expected value of quadratic forms of state and control vectors, (b) quadratic forms of the expected value of the state vector, and (c) the linear component of the expected value of the state vector. The purpose of the present paper is to design optimal control strategies that are independent of distributional assumptions on the stochastic parameters and subject to hard constraints on the input manipulated variables and to provide a numerically tractable algorithm for practical applications. All expressions are presented in terms of the first- and second-order conditional moments. The results are applied to a problem of investment portfolio optimization with serially correlated returns. We present the numerical modelling results, based on stocks traded on the Russian Stock Exchanges MICEX.

1. Introduction

Lately, there has been a steadily growing need and interest in systems with stochastic parameters and/or multiplicative noise. The same systems have been gaining greater acceptance in many engineering applications. Financial engineering is also an important field of application where such models are used for describing the evolution of investment portfolios (see, for instance, Costa and Araujo (2008), Dombrovskii and Lyashenko (2003), Hu and Zhou (2005)).

Several results related to control systems with stochastic parameters subject to constraints have already been derived. In recent years, considerable interest has been focused on model predictive control (MPC), also known as receding horizon control (RHC). MPC proved to be an appropriate and effective technique to solve the dynamic control problems subject to input and state/output constraints.

MPC for constrained discrete-time linear systems with random parameters and/or multiplicative noises has been intensively studied lately. Some of the recent works on this subject can be found, for instance, in Bemporad and Di Cairano (2011), Bernardini and Bemporad (2009), Calafiore and Fagiano (2013), Cannon, Kouvaritakis, and Wu (2009), Dombrovskii, Dombrovskii, and Lyashenko (2005, 2006), Dombrovskii and Obyedko (2011), Lee and Cooly (1998), and Primbs (2009).

In particular, Lee and Cooly (1998) investigate systems with independent and identically distributed parameters, while Dombrovskii et al. (2005) study systems with both control and state multiplicative noises and stochastic independent parameters under hard constraints on input variables. Cannon et al. (2009) study systems with control and state multiplicative noises where constraints are assumed to be soft and probabilistic. Primbs and Sung (2009) study systems with control and state multiplicative noises in the presence of soft quadratic expectation constraints. In Dombrovskii et al. (2006), MPC of linear systems with random dependent parameters, where the evolution of parameters is described by linear difference stochastic equations under hard constraints on the control variables, is considered. Dombrovskii and Obyedko (2011) investigate the MPC problem of discrete-time Markov jump linear systems with multiplicative noise subject to hard constraints on the control variables. Related results in MPC design via scenario generation can be found in Bemporad and Di Cairano (2011), Bernardini and Bemporad (2009), and Calafiore and Fagiano (2013). Note that the scenario-based MPC is often computationally demanding and assumes a specific probability distribution for the model parameters.

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In this paper, we consider MPC for constrained discrete-time systems with stochastic parameters which are assumed to be a set of serially correlated time series. The lead–lag relationships between component series are described by the matrices of the second-order conditional moments and the knowledge of the statistical distribution of the parameters is not assumed.

We consider a generalized performance criterion which is composed of a weighted sum of a linear combination of the (a) expected value of quadratic forms of state and control vectors, (b) quadratic forms of the expected value of the state vector, and (c) a linear part in the expected value of the state vector. The motivation for adopting this type of criterion is that in several situations we provide solutions for two special cases. The first one is MPC for the quadratic criterion and the second MPC for the mean–variance criterion. Note that this cost function is not traditionally used in MPC theory. This approach to cost function formulation is based on an idea proposed in Costa and Araujo (2008) that a generalized multi-period mean–variance portfolio selection problem is considered without constraints and with a finite horizon.

The main goal of the present paper is to design optimal control strategies that are independent of distributional assumptions on the stochastic parameters and are subject to hard constraints on the input manipulated variables and to provide a numerically tractable algorithm for practical applications. We derive an exact expression for the predicted performance criterion as an explicit function of predicted input variables that can be optimized online by minimizing over the vector of predicted input variables subject to hard constraints.

The results are applied to a problem of investment portfolio optimization with serially correlated returns. Note that the portfolio management problem is the key problem of financial engineering that includes a set of major problems associated with the control of complex dynamic systems with stochastic parameters under constraints. Therefore, the investment portfolio can be a powerful platform for testing the effectiveness of the designed control strategies.

There are many examples of the MPC in finance applications. Some recent works can be found in Bemporad, Puglia, and Gabbriellini (2011), Dombrovskii, Dombrovskii, and Lyashenko (2004), Dombrovskii et al. (2005, 2006), Dombrovskii and Obyedko (2011), Herzog, Dondi, and Geering (2007), and Primbs (2009). In all of these papers, authors assume the hypothesis of serially independent returns and/or consider the explicit form of the model describing the price process of the risky assets (e.g., geometric Brownian motion, etc.).

Related results in multi-period portfolio optimization can be found in Calafiore (2008, 2009) where a multi-stage optimization model is developed. In a developed model portfolio, diversity constraints are imposed in expectation (soft constraints). Calafiore (2008, 2009) in his works has proposed the use of a linearly parameterized class of feedback control policies that are affine functions of the past return innovations. However, it is difficult to obtain an exact analytic formulation of the optimization problem based on the proposed model for the case of generic and serially correlated return processes. Calafiore (2008, 2009) proposed an approximated technique to solve the problem via stochastic simulations of the return series that can be used in practice when a full stochastic model for return dynamics is available.

In this paper, we propose a framework for the computation of dynamic trading strategies subject to serially correlated returns and hard constraints on the trading amounts. The only conditions imposed on the distributions of the asset returns are the existences of the conditional mean vectors and of the conditional second-order moments. No assumptions about the correlation structure between different time points or about the distribution of the asset returns are needed. The proposed trading strategies are convenient to adaptive implementation. Adaptive algorithms have the ability to adapt to the underlying data by dynamically incorporating new information into the decision process and they are more suitable for non-stationary environments, such as those in finance. The motivation is within the context of algorithmic trading, which demands fast and recursive updates of portfolio allocations as new data arrives.

We want to demonstrate the performance of our model under real market conditions. We present the numerical modelling results, based on stocks traded on the Russian Stock Exchanges MICEX.

2. Problem formulation

We consider the following discrete-time system with stochastic parameters on the probabilistic space $(\Omega, \mathfrak{F}, \mathbf{P})$.

$$x(k+1) = Ax(k) + B[\eta(k+1), k+1]u(k), \tag{1}$$

where $x(k) \in \mathbb{R}^{n_x}$ is the vector of state, $u(k) \in \mathbb{R}^{n_u}$ is the vector of control inputs, and $\eta(k) \in \mathbb{R}^q$ is assumed to be a stochastic time series. The matrices $A \in \mathbb{R}^{n_x \times n_x}$, $B[\eta(k), k] \in \mathbb{R}^{n_x \times n_u}$ are the system matrix and the input matrix, respectively. All of the elements of $B[\eta(k), k]$ are assumed to be linear functions of $\eta(k)$.

Let $\mathbb{F} = (\mathfrak{F}_k)_{k \geq 1}$ be the complete filtration with σ-field $\mathfrak{F}_k$ generated by the $\{\eta(s) : s = 0, 1, 2, \ldots, k\}$ that models the flow of information to time k.

Throughout the paper, we use the following notations. We denote with $E\{a/b\}$ the conditional expectation of a with respect to b. For any matrix $\psi[\eta(k+i), k+i]$, dependent on $\eta(k+i)$, $\overline{\psi}(k+i) = E\{\psi[\eta(k+i), k+i]/\mathfrak{F}_k\}$, $\tilde{\psi}(k+i) = \psi(k+i) - \overline{\psi}(k+i)$, $i \geq 1$, without indicating the explicit dependence of matrices on $\eta(k+i)$. Additionally, we use the standard notation, for square matrix M, $M \geq 0 \, (M > 0)$ to denote that the matrix M is positive semidefinite (positive definite).

We allow the time series $\eta(k)$ to be serially correlated. Let us assume that we know the first- and second-order conditional moments for the stochastic vector $\eta(k)$ about $\mathfrak{F}_k$:

$$E\{\eta(k+i)/\mathfrak{F}_k\} = \overline{\eta}(k+i),$$
$$E\left\{\eta(k+i)\eta^T(k+j)/\mathfrak{F}_k\right\} = \Theta_{ij}(k), \quad (i, j = 1, 2, \ldots, l).$$

Therefore, the lead–lag relationships between component series $\eta_t(k+i)$ and $\eta_f(k+j)$ are described by the matrices $\Theta_{ij}(k)$ of the second-order conditional moments.

We impose the following inequality constraints on the control inputs (element-wise inequality)

$$u_{\min}(k) \leq S(k)u(k) \leq u_{\max}(k), \tag{2}$$

where $S(k) \in \mathbb{R}^{p \times n_u}$; $u_{\min}(k), u_{\max}(k) \in \mathbb{R}^p$.

We use the MPC methodology in order to define the optimal control strategy. The main concept of MPC is to solve an open-loop constrained optimization problem at each time instant and to implement only the initial optimizing control action of the solution.

We define the following cost function with receding horizon, which is to be minimized at every time k,

$$\begin{aligned} J(k+m/k) &= \sum_{i=1}^{m} E\{x^T(k+i)R_1(k+i)x(k+i)/x(k), \mathfrak{F}_k\} \\ &- \sum_{i=1}^{m} E\{x^T(k+i)/x(k), \mathfrak{F}_k\}R_2(k+i)E\{x(k+i)/x(k), \mathfrak{F}_k\} \\ &- \sum_{i=1}^{m} R_3(k+i)E\{x(k+i)/x(k), \mathfrak{F}_k\} \\ &+ \sum_{i=0}^{m-1} E\{u^T(k+i/k)R(k+i)u(k+i/k)/x(k), \mathfrak{F}_k\}, \end{aligned} \tag{3}$$