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Experimental characterization of veering crossing and lock-in in simple mechanical systems

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ABSTRACT

In this paper, mode veering, crossing and lock-in phenomena are experimentally analyzed and characterized. Their occurrence is generally found, under different conditions, when there is a parameter variation in the system that produces a change in its behaviour. It often happens that, when the natural frequencies of two modes approach each other, they can cross, veer and eventually present a lock-in state. The problem is analytically investigated for general weakly-coupled two-degrees of freedom systems and experiments, appropriately designed to highlight these phenomena, are presented. In particular, experimental evidence of the damping-dependent transition from veering to crossing is investigated for a two beam system, and experimental lock-in is recalled to show how the gyroscopic systems become unstable when two coupled mechanical parts have the same eigenvalue.

1. Introduction

In real structures, uncertainties, irregularities or variable operative conditions may affect significantly the structure behaviour. This problem is widely considered in the literature either theoretically or experimentally. Here we examine under which conditions particular types of coupling between the system degrees of freedom (elastic, inertial or gyroscopic) lead to eigenvalues interaction, producing often interesting and unexpected phenomena. We refer specifically to three main situations, *i.e.* veering, crossing and mode lock-in, that are primarily related to the interacting modes but become a singular signature of the whole system.

Variations in the geometric and physical parameters (*e.g.* stiffness, mass, friction coefficients, etc.) due to design variations, wear-off or operational conditions change the system eigenvalues. Occasionally, a pair of eigenvalues can approach each other and, when this happens, small variations in such parameters can lead to dramatic changes in the whole system dynamics. For example, small variations in gyroscopic terms may induce the transition between mode veering and mode lock-in in a brake system, causing squeal noise.

Mode veering is a common phenomenon associated with the eigenvalue locus: due to the variation of one or more parameters, two modes may approach each other but, instead of crossing, they veer away and finally diverge. The outstanding observation is that, after veering, the two corresponding eigenvectors interchange, *i.e.* the direction of the eigenvalue locus of one mode corresponds to the direction of the second one before veering.

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Mode crossing can be considered a particular and quite unusual case of veering. In linear conservative and non gyroscopic systems, the weaker is the elastic coupling between the degrees of freedom, the smaller is the range of parameters where veering occurs. When the off-diagonal terms of the stiffness matrix are zero, it is possible that the two uncoupled systems have the same frequency and, thus, crossing is observed, *i.e.* the natural frequencies of two modes exchange their position in frequency. In this case, at the crossing point, the two modes are not uniquely defined, and can be described as the resultant of the two independent eigenvectors approaching the crossing point.

Moreover, crossing may occur in mechanical systems in case a varying parameter does not affect one or more modes of the system: e.g. an hinged–hinged beam with a varying stiffness located in the centre.

More generally, crossing can be observed in non conservative systems. In fact, in the presence of damping, two modes can have the same natural frequency though maintaining their individual mode shapes.

Mode lock-in is a phenomenon mirror to veering, that can be observed when gyroscopic terms are present. Due to the variation of one or more parameters, the frequencies of two modes approach more and more but, instead of veering away, they are attracted to one another and coalesce, leading to an unstable behaviour.

If one considers three similar systems, characterized by veering, crossing and lock-in, respectively, far from a critical zone, that is characterized by close eigenvalues, the three systems have similar dynamics, while, inside this zone, the behaviours show quite large qualitative differences. It is important to stress here that the extent of the critical zone depends on the coupling between the DOFs of the systems.

Several works in the literature show the experimental evidence of these phenomena. Veering is certainly the one by far the most investigated. After some preliminary works on beams by Petyt and Fleisxer [1], and on curved plates by Nair and Durvarsula [2], Leissa [3] gave a detailed description of this phenomenon and named it veering. In [4], Perkins and Mote formulated a first analytical criterion for veering, while Pierre [5] developed a perturbation technique to show that the occurrence of strong mode localization and eigenvalue loci veering are manifestations of the same phenomenon, often caused by small structural irregularities in systems with close eigenvalues. Other theoretical and numerical works on veering are due to Balmes [6], du Bois et al. [7], Bonisoli et al. [8]. Vidoli and Vestroni [9] gave recently a geometrical formulation of veering, and some experimental evidence of it are presented in [9,10]. In [9] the case of a plate embedded with a set of piezoelectric actuators is considered, while in [10] Lin and Parker discuss the onset of veering in planetary gears. Very recently Mace and Marconi made a clear picture of the veering phenomenon in weakly coupled systems [11] ad addressed the case of veering and lock-in for continuous undamped systems [12].

Crossing has less evidence in literature, although any theoretical work on veering discusses in principle the crossing phenomenon (*e.g.* [4–9]). However in the authors' knowledge there are not specific descriptions of experimental crossing phenomena, excluding those that were appropriately designed to show this possibility (see *e.g.* [7]) or when dealing with independent mode shapes with no coupling.

Mode lock-in (also called modal coupling instability or flutter instability) was particularly observed in flutter phenomena and several problems of aeroelasticity (*e.g.* [13]), and, more recently, in brake squeal, in several types of problems of (especially dry) contact between solid bodies [14] and in gyroscopic systems with negative-definite stiffness matrices [15]. Considering specifically contact problems, mode lock-in was initially highlighted in the beam-on disk set-up, developed by Akay et al. to investigate friction at interfaces [16]. More recent studies on brake squeal noise with simplified lab set-ups have always shown that squeal noise is an instability condition, that is reached when two eigenfrequencies of the system, due to the asymmetry of the stiffness matrix caused by friction forces, coalesce and become unstable [17–20].

Several works study the effect of damping on the lock-instability. In brake squeal literature, Hoffmann et al. [21] found that the presence of damping causes imperfect bifurcation with unstable eigenvalue, Sinou et al. [22–24] study the stability of a brake system taking into account the destabilizing effect on damping. Massi and Giannini [20] measured on the beam on disc set-up the lock-in instability and the extent of the unstable zone and found an experimental validation of the previous findings. Kirillov in [25] addressed the effect of damping in gyroscopic systems, while in [26] addressed a comprehensive analytical study on the interaction of eigenvalues of generic matrices relating the effect of several parameters, including damping, on the veering and the lock-in characteristics.

Goal of this paper is to show and discuss under which conditions veering, crossing and mode lock-in can be observed, which parameters influence transition from one phenomenon to the other, and to show some experimental occurrences of these dynamic behaviours even in quite simple mechanical systems.

2. Analytical developments

2.1. Veering

Let us first consider the rather simple two degrees of freedom undamped system, depicted in Fig. 1.