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Stochastic dynamic response and reliability assessment of controlled structures with fractional derivative model of viscoelastic dampers

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ABSTRACT

Viscoelastic dampers, where fractional derivatives are involved, are often considered for use to mitigate dynamic response of structures. However, it is not an easy task to obtain the probabilistic dynamic response and the reliability of controlled structures with fractional terms. For this purpose, an efficient methodology based on the probability density evolution method is proposed, where the generalized density evolution equation is present to capture the instantaneous probabilistic dynamic response and the dynamic reliability can be evaluated from the standpoint of probability dissipation. Numerical solution is of practical necessity, where a deterministic procedure to solve the equation of motion with fractional derivatives is embedded. Therefore, the precise integration method (PIM) is extended to numerically integrate the equation of motion with fractional terms, which offers high accuracy. The numerical results verify the effectiveness of the advocated methodology, but also indicate the viscoelastic dampers can enhance the seismic performance of structures significantly.

1. Introduction

Dynamic response and reliability analysis of structures with dampers described by fractional models, where the randomness in both structural parameters and external excitations are involved, can provide a comprehensive and thorough understanding of the controlled structural performance. To this end, the equation of motion of controlled structures, which not only involves fractional derivatives and regular derivatives when the fractional viscoelastic dampers are installed to structures, but also relates to the randomness ubiquitous in engineering practice, is of great concern.

The work relative to this subject can be investigated analytically and statistical characteristics are derived. A fractional calculus approach is adopted to model the Brownian motion, leading to the fractional Langevin equation solved by Laplace transform [1]. At the same time, a frequency domain related approach is put forward for random vibration analysis with fractional dampers by Spanos and Zeldin [2], such approach is applied to a white noise excited single-degree-of-freedom system with a tuned mass damper where a viscoelastic damping element is involved [3]. For some special fractional order, such as the order of 1/2, an analytical theme using eigenvector expansion method and the properties of Laplace transforms

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of convolution integrals is derived for stochastic dynamic systems containing fractional derivatives in damping behaviors [4]. After that, stochastic seismic response of a structure with viscoelastic dampers is studied by using the Fourier-transform based technique and the Duhamel integral-type expression [5]. A similar expression is also conducted to study the statistical behaviors of a stochastic dynamic system with arbitrary fractional order [6]. Moreover, a stochastic averaging procedure for a single-degree-of-freedom (SDOF) strongly nonlinear system with light damping modeled by a fractional derivative model under Guassian white noise excitations is developed by using the so-called generalized harmonic functions [7]. Di Matteo et al. derive an approximate closed-form solution for determining the non-stationary response probability density function (PDF) of randomly excited linear and nonlinear oscillators endowed with fractional derivatives elements by using the Wiener path integral in conjunction with a variational formulation [8]. Compared to the analytical solutions, only a handful of papers are devoted to numerical methods to obtain the stochastic dynamic response of systems including fractional derivatives. The easiest way to perform the numerical solution is the Monte Carlo simulation(MCS). Spanos and Evangelatos implement MCS to compare the standard deviation with that derived by statistical linearization when fractional damping is concerned [9]. A methodology based on the combination of modified Euler method and Monte Carlo method is put forward to numerically investigate the statistical behaviors of such problems involving fractional derivatives [6].

Nevertheless, stochastic dynamic analysis of a general linear/nonlinear multiple -degree-of-freedom (MDOF) structure with fractional derivatives subjected to general non-stationary, non-Guassian stochastic excitations may have not been reported, let alone the corresponding dynamic reliability assessment. The aim of this work is to present a new methodology based on the probability density evolution method (PDEM) together with a new time-domain numerical solution to the equation of motion with fractional derivatives to efficiently perform the stochastic dynamic response and reliability analysis of structures with dampers of fractional models.

This paper is organized as follows: in Section 2, the basic definitions and preliminaries of fractional derivatives for viscoelastic dampers are presented. In Section 3, the framework of the stochastic dynamic response and reliability analysis of controlled structures with fractional derivatives is proposed on the basis of PDEM. Section 4 is devoted to extending an effective deterministic numerical procedure, which is embedded in a stochastic dynamic analysis, to solve the equation of motion involving fractional terms with high accuracy. Numerical simulations are performed in Section 5 to validate the effectiveness of the proposed methodology. Finally, some concluding remarks are reported in Section 6.

2. Fractional models for viscoelastic dampers

In civil engineering, viscoelastic dampers, which are some preferred energy dissipation devices for passive response control, are successfully applied to reduce any excessive vibrations of buildings caused by winds and earthquakes [10,11]. It is now recognized that the fractional derivative model for the rheological property of viscoelastic damper can accurately capture the stiffness and energy dissipation characteristics, which are of great interest in dynamic analysis [12]. In addition, the most attractive feature is that the fractional model is capable of describing the mechanical behavior of viscoelastic damper using a few model parameters.

2.1. Fundamentals of fractional derivative

The term of fractional derivative is a generalization of the ordinary differential to non-integer (arbitrary) order. Three famous definitions for fractional derivative are: the *Riemann–Liouville* (*RL*) definition, the *Caputo* (*C*) definition and the *Grunward–Letnikov* (*GL*) definition [13–15].

For numerical calculation, the *GL* definition is adopted mostly, which is defined as

$$D_t^\alpha \langle f(t) \rangle = \lim_{\Delta t \to 0^+} \frac{1}{(\Delta t)^\alpha} \sum_{j=0}^{[(t-a)/\Delta t]} (-1)^j \binom{\alpha}{j} f(t - j\Delta t) \tag{1}$$

where $0 < \alpha < 1$ is the fractional order, $D_t^\alpha \langle . \rangle = d^\alpha / dt^\alpha$ is the fractional operator, Δt is the time step and a is the starting time, $[A]$ denotes the integer operator, which yields to the integer part of A, for example, $[20.12] = 20$.

Let $w_j^\alpha = (-1)^j \binom{\alpha}{j}$, which can be evaluated by

$$w_0^\alpha = 1, \quad w_j^\alpha = \left(1 - \frac{\alpha+1}{j}\right) w_{j-1}^\alpha, j = 1, 2, \ldots \tag{2}$$

and $t = a + n\Delta t, f(t - j\Delta t) = f_{n-j}$, then Eq. (1) can be rewritten as

$$D_t^\alpha \langle f(t) \rangle = \lim_{\Delta t \to 0^+} \frac{1}{(\Delta t)^\alpha} \sum_{j=0}^{n} w_j^\alpha f_{n-j} \tag{3}$$