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On equivalence of two stability criteria for continuous-time switched systems with dwell time constraint[☆]

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ABSTRACT

In this note, a study on the equivalence of two stability criteria for continuous-time switched linear systems with dwell time constraint is presented. It is demonstrated that, for any dwell time satisfying the conditions in stability criterion proposed in Geromel and Colaneri (2006), the conditions in stability criterion of Allerhand and Shaked (2011) also hold as long as the number of decision variables and related LMIs is sufficiently large, which implies the two stability criteria are intrinsically equivalent. The equivalence is obtained by showing that two criteria can be derived from each other with a sufficiently large number of decision variables and LMIs. A numerical example is proposed to illustrate the theoretical results, and an extension to uncertain case is briefly presented.

1. Introduction

In this brief note, we consider the continuous-time switched linear system in the form of

$$\begin{aligned} \dot{x}(t) &= A_{\sigma(t)}x(t) \\ x(0) &= x_0 \end{aligned} \tag{1}$$

where $x(t), x_0 \in \mathbb{R}^n$ are the state of the system and the initial condition, respectively. Define index set $\mathcal{I} =: \{1, 2, \ldots, N\}$ where N is the number of subsystems. $\sigma(t) : [0, \infty) \to \mathcal{I}$ denotes the switching signal function, which is assumed to be a piecewise constant function continuous from right. Let the discontinuity points of $\sigma(t)$ be denoted by t_n, and let t_0 stand for the initial time by convention, the switching sequence can be described as $\mathcal{S} := \{t_n\}_{n\in\mathbb{N}}$. Calling $\mathcal{D}_\tau$ the set of all switching policies with dwell time τ, that is the set of all $\sigma(t)$ for which the time interval between successive discontinuities of $\sigma(t)$ satisfies $t_{n+1} - t_n \geq \tau,\ \forall n \in \mathbb{N}$. Only *non-Zeno* switchings are considered in this work, i.e., only finitely many switches can occur during any finite interval.

Generally, the stability and stabilization problems are the main concerns in the field of switched systems, which have been extensively studied in the literature (Decarlo, Branicky, Pettersson, & Lennartson, 2000; Lin & Antsaklis, 2009; Margaliot, 2006; Shorten, Wirth, Mason, Wulff, & King, 2007). By constraining the switching law via dwell time (or average dwell time), the combination of multiple Lyapunov function (MLF) technique and dwell time approach is an effective method dealing with stability problems for switched systems (Allerhand & Shaked, 2011; Briat, 2014; Chesi, Colaneri, Geromel, Middleton, & Shorten, 2012; Geromel & Colaneri, 2006a,b; Hespanha, Liberzon, & Morse, 1999; Xiang & Xiao, 2014a,b). In Geromel and Colaneri (2006a), a stability criterion involving the exponential term of system matrices, i.e. $e^{A_i\tau}$, is proposed, which could be the best stability result attained so far for the nominal case. The interest for the approach in Geromel and Colaneri (2006a) lies in the monotonically decreasing of the Lyapunov function along with the time instant sequence $\{t_n\}_{n\in\mathbb{N}}$. However, though this method is simple and efficiently computable, it is difficult to be further generalized to uncertain systems due to the difficulty of considering uncertainties at the exponential. Thus, in a recent result (Allerhand & Shaked, 2011), the idea of time-scheduled Lyapunov function is employed to obtain a stability criterion which is convex in system matrices. This newly derived result is convenient to be generalized due to its convex feature.

Moreover, through the numerical simulation, e.g. the Example 1 in Allerhand and Shaked (2011), it is interesting to observe that the result by Allerhand and Shaked (2011) tends to the one obtained by Geromel and Colaneri (2006a) as the number of decision variables and related linear matrix inequality (LMI) conditions are increased, so a question about the relationship between two stability criteria naturally arises here: *Can the result*

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in Allerhand and Shaked (2011) be equivalent to the result in Geromel and Colaneri (2006a) as long as the number of decision variables and related LMI conditions are sufficiently large for Allerhand and Shaked (2011)? This fact seems true by observing the simulation results in Allerhand and Shaked (2011), however, only a numerical example is obviously not enough to justify the equivalence. Therefore, motivated by the question, the main aim of this note is to reveal the equivalence of two stability criteria by showing that, for any dwell time satisfying stability criterion of Geromel and Colaneri (2006a), there always exist a sufficient large number of decision variables and related LMIs such that the conditions in stability criterion of Allerhand and Shaked (2011) also hold.

The concerned two stability criteria and problem formulation are introduced in Section 2. The main results on studying the equivalence are presented in Section 3, showing that the two criteria can be derived from each other. Conclusions are given in Section 4.

2. Problem formulation

In the following, we will make extensive uses of the matrix expressions:

$$\mathscr{L}(A,P) := A^\top P + PA$$

$$\mathscr{E}(A,P,Q,\tau) := e^{A^\top \tau} P e^{A\tau} - Q$$

$$\mathscr{D}_1(A,P,Q,\delta) := A^\top P + PA + \frac{P-Q}{\delta}$$

$$\mathscr{D}_2(A,P,Q,\delta) := A^\top Q + PQ + \frac{P-Q}{\delta}$$

$$\mathscr{D}(A,\mathscr{P}(t)) := A^\top \mathscr{P}(t) + \mathscr{P}(t)A + \frac{\mathrm{d}\mathscr{P}(t)}{\mathrm{d}t}.$$

The two stability criteria concerned in this note are given in the following. In Geromel and Colaneri (2006a), a simple sufficient condition for the stability of switched system (1) is introduced.

Lemma 1 (*Geromel & Colaneri, 2006a*). *Given that for some positive scalar τ, there exists a collection of symmetric matrices $P_i \succ 0,\ i = 1,2,\ldots,N$ of compatible dimensions that satisfy the following*

$$\mathscr{L}(A_i,P_i) \prec 0, \quad \forall i \in \mathscr{I} \tag{2}$$

$$\mathscr{E}(A_i,P_j,P_i,\tau) \prec 0, \quad \forall i,j \in \mathscr{I},\ i \neq j. \tag{3}$$

Then the system (1) is globally asymptotically stable with any switching law $\sigma(t) \in \mathcal{D}_\tau$.

The Condition (3) stems from the requirement that switching from the ith subsystem to the jth one, the value of the Lyapunov function $V(t) = x^\top(t)P_\sigma(t)x(t)$ just after switching is less than its value at the instant when the ith subsystem became active. An alternative way to guarantee the decrease of $V(t)$ is to require that the value of $V(t)$, τ seconds after the switching, is less than the value it had just prior to the switching. Thus, an alternative result equivalent to Lemma 1 is given by just swapping the matrices P_i and P_j in $\mathscr{E}(A_i,P_j,P_i,\tau)$, see Lemma 2 in Allerhand and Shaked (2011) by considering the nominal case.

Lemma 2 (*Allerhand & Shaked, 2011*). *Given that for some positive scalar τ, there exists a collection of symmetric matrices $P_i \succ 0,\ i = 1,2,\ldots,N$ of compatible dimensions that satisfy the following*

$$\mathscr{L}(A_i,P_i) \prec 0, \quad \forall i \in \mathscr{I} \tag{4}$$

$$\mathscr{E}(A_i,P_i,P_j,\tau) \prec 0, \quad \forall i,j \in \mathscr{I},\ i \neq j. \tag{5}$$

Then the system (1) is globally asymptotically stable with any switching law $\sigma(t) \in \mathcal{D}_\tau$.

In terms of Lyapunov functions with quadratic structure, the above results seem to be the best possible. However, the LMIs in (3) and (5) depend on the exponential term $e^{A_i\tau}$, which is not convex in A_i. This prevents further extensions such as to robust dwell time characterization, essentially due to the difficulty of considering uncertainties at the exponential. Thus, an alternative result in terms of LMIs which are affine in the system matrices is proposed in Allerhand and Shaked (2011).

Lemma 3 (*Allerhand & Shaked, 2011*). *The system (1) is globally asymptotically stable with a switching law $\sigma(t) \in \mathcal{D}_\tau$ if there exist a collection of symmetric matrices $P_{i,k} \succ 0,\ i = 1,2,\ldots,N,\ k = 0,\ldots,K$ of compatible dimensions, where $K \in \mathbb{N}\backslash\{0\}$ may be chosen a priori, according to the allowed computational complexity, and a sequence $\{\delta_k\}_{k=1,\ldots,K}$, where $\delta_k > 0$ and $\sum_{k=1}^{K}\delta_k = \tau$ such that for all $i \in \mathscr{I}$ the following holds:*

$$\mathscr{D}_1(A_i,P_{i,k},P_{i,k+1},\delta_{k+1}) \prec 0, \quad k = 0,\ldots,K-1 \tag{6}$$

$$\mathscr{D}_2(A_i,P_{i,k},P_{i,k+1},\delta_{k+1}) \prec 0, \quad k = 0,\ldots,K-1 \tag{7}$$

$$\mathscr{L}(A_i,P_{i,K}) \prec 0 \tag{8}$$

$$P_{j,0} - P_{i,K} \prec 0, \quad j \in \mathscr{I},\ j \neq i. \tag{9}$$

Let $\tau(K) : \mathbb{N}\backslash\{0\} \to [0,\infty)$ map each $K \in \mathbb{N}\backslash\{0\}$ with minimal dwell time by Lemma 3. In Allerhand and Shaked (2011), it shows the value of $\tau(K)$ is closely related to the choice of K. By the numerical comparison made in Allerhand and Shaked (2011), see Example 1 in Allerhand and Shaked (2011), it shows that Lemma 3 is more conservative than Lemma 1 or 2 when K is chosen to be small, but the result of Lemma 3 tends to the one of Geromel and Colaneri (2006a) when K is increased, and finally the two results seem to become equivalent for a certain K. Thus, by observing the simulation results in Allerhand and Shaked (2011), one would naturally question: Can Lemma 3 be equivalent to Lemmas 1 and 2, if K is chosen to be sufficiently large? This question can be stated as follows.

Question 1. *Given any dwell time τ^* such that Lemma 1 or 2 holds, can we always find a $K^* \in \mathbb{N}\backslash\{0\}$ for Lemma 3 such that $\tau(K) = \tau^*$, $\forall K \geq K^*$?*

The answer to this question is the main concern in this note, if there always exists such a $K^* \in \mathbb{N}\backslash\{0\}$ for any dwell time, it can be claimed that the two stability criteria are intrinsically equivalent when the K is chosen sufficiently large, i.e. $K \geq K^*$, in Lemma 3. The rest of this note is to answer the above question, by showing the two stability criteria can be derived from each other as long as a sufficiently large K is chosen.

3. Main results

The key idea of Lemma 3 relies in the time-scheduled Lyapunov function in the form of $V(t) = x^\top(k)\mathscr{P}(t)x(t)$. Thus, before presenting the main result, a preliminary result for time-varying matrix $\mathscr{P}(t)$ is given.

Lemma 4. *For a matrix A, there always exist a scalar $\delta^* > 0$ and matrices $P_0 \succ 0,\ P_1 \succ 0$ such that*

$$\mathscr{D}(A,\mathscr{P}(t)) \prec 0, \quad t \in [t_0, t_0+\delta) \tag{10}$$

where $0 < \delta \leq \delta^*$ and $\mathscr{P}(t)$ defined by

$$\mathscr{P}(t) = P_0 + (P_1 - P_0)\frac{t-t_0}{\delta}, \quad t \in [t_0, t_0+\delta). \tag{11}$$

Proof. By the structure of $\mathscr{P}(t)$ in (11), we have

$$\mathscr{D}(A,\mathscr{P}(t)) = \alpha_1\mathscr{D}_1(A,P_0,P_1,\delta) + \alpha_2\mathscr{D}_2(A,P_0,P_1,\delta) \tag{12}$$

where $\alpha_1 = 1 - (t-t_0)/\delta$ and $\alpha_2 = (t-t_0)/\delta$. Thus, (10) is equivalent to

$$\mathscr{D}_1(A,P_0,P_1,\delta) \prec 0 \tag{13}$$

$$\mathscr{D}_2(A,P_0,P_1,\delta) \prec 0. \tag{14}$$

Thus, the existence of $\mathscr{P}(t)$ satisfying (10) can be converted to the existence of P_0 and P_1 satisfying (13) and (14).