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Nonsingular fixed-time consensus tracking for second-order multi-agent networks[☆]

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ABSTRACT

This paper investigates the fixed-time consensus tracking problem for second-order multi-agent systems in networks with directed topology. Global well-defined nonlinear consensus protocols are constructed with the aid of a newly-designed sliding surface for each double-integrator agent dynamics. In particular, the proposed framework eliminates the singularity and the settling time is assignable for any initial conditions. This makes it possible for network consensus problems to design and estimate the convergence time off-line. Finally, simulation is included to demonstrate the performance of the new protocols.

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1. Introduction

Network Consensus (Ren, 2008) means that a group of agents reaches an agreement upon a common value by local interaction. In some applications, groups of agents are required to track a dynamic leader (time-varying reference information), which is called *consensus tracking*. In the analysis of consensus problems, an important performance index for a proposed consensus protocol is convergence rate (Wang & Xiao, 2010). Olfati-Saber and Murray (2004) proposed a linear consensus protocol and demonstrated that the *algebraic connectivity* of a interaction graph qualified the convergence rate. On the other hand, *finite-time consensus* problem has been promoted to achieve high-speed convergence. The benchmark work due to Bhat and Bernstein (2000) related the regularity properties of the Lyapunov functions to the settling time function in finite-time stability analysis for autonomous systems. Xiao and Wang (2007) and Xiao, Wang, and Chen (2011) expanded the finite-time control idea (Bhat & Bernstein, 2000) to multi-agent systems with single-integrator kinematics. Chen, Cao, and

Ren (2012) and Cortés (2006) proposed discontinuous finite-time protocols for first-order agent networks based on different ways of applying the signum function. However, it is nontrivial to extend the finite-time consensus algorithm from the first-order case to the second-order case straightforwardly. Despite the difficulties, still some progress on this issue has been made recently. The works due to Wang and Hong (2008) and Zhao, Duan, Wen, and Zhang (2013) presented the finite-time consensus and tracking control respectively for second-order multi-agent systems via using homogeneity with dilation. Khoo, Xie, and Man (2009) expanded the terminal sliding mode technique (Feng, Yu, & Man, 2002) to the finite-time consensus problem of second-order multi-robot systems. The work due to Li, Du, and Lin (2011) proposed continuous finite-time consensus algorithms for leaderless and leader-follower second-order multi-agent systems respectively by adding a power integrator method.

However, the settling time functions derived in the finite-time consensus depend on initial states of the agents, which prohibits their practical applications if the knowledge of initial conditions is unavailable in advance. Recently, Zuo and Tie (2014a,b) present a novel class of nonlinear consensus protocols for single-integrator multi-agent networks, called fixed-time consensus which assumes uniform boundedness of a settling time regardless of the initial conditions, i.e. fixed-time stability (Polyakov, 2012). Unfortunately, the direct extension to second-order case has come across a challenging issue: the singularity in the controller (Feng et al., 2002; Feng, Yu, & Man, 2013; Wu, Yu, & Man, 1998). Motivated

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by this, this paper extends the idea in Zuo and Tie (2014a,b) and constructs global well-defined fixed-time consensus protocols for second-order multi-agent networks with directed information flow. The key contributions include: (i) a guaranteed settling time independent of initial conditions is obtained, i.e., fixed-time consensus; (ii) a continuous sinusoid function is introduced into the protocols to eliminate the singularity. To the author's knowledge, no results on fixed-time consensus tracking with assignable settling time for second-order multi-agent networks are available till now.

2. Preliminaries

2.1. Graph theory notions

A weighted graph $\mathcal{G}(A) = \{\mathcal{V}, \mathcal{E}, A\}$ consists of a node set $\mathcal{V}(\mathcal{G}) = \{\pi_1, \pi_2, \dots, \pi_n\}$, an edge set $\mathcal{E}(\mathcal{G}) \subseteq \mathcal{V} \times \mathcal{V}$ and an adjacent matrix $A = [a_{ij}] \in \mathbb{R}^{n \times n}$. An edge (π_i, π_j) on $\mathcal{G}(A)$ denotes the state of node π_i is available to node π_j , but not necessarily vice versa. If $(\pi_i, \pi_j) \in \mathcal{E}$, then node π_i is called a *neighbor* of node π_j . The index set of all neighbors of π_j is denoted by $\mathcal{N}_j = \{i : (\pi_i, \pi_j) \in \mathcal{E}\}$. The weighted adjacent matrix A of a directed graph is defined such that $a_{ij} = 1$ for $(\pi_j, \pi_i) \in \mathcal{E}$ and $a_{ij} = 0$ otherwise. Let $L = [l_{ij}] \in \mathbb{R}^{n \times n}$ be the graph Laplacian of \mathcal{G} . A graph has a directed spanning tree if a subset of the edges forms a spanning tree (Ren & Beard, 2005).

2.2. Some lemmas

Lemma 1 (Zuo & Tie, 2014a). Let $\xi_1, \xi_2, \dots, \xi_N \geq 0$. Then

$$\sum_{i=1}^N \xi_i^p \geq \left(\sum_{i=1}^N \xi_i \right)^p \quad \text{if } 0 < p \leq 1 \quad (1)$$

$$\sum_{i=1}^N \xi_i^p \geq N^{1-p} \left(\sum_{i=1}^N \xi_i \right)^p \quad \text{if } 1 < p < \infty. \quad (2)$$

Lemma 2 (Zuo & Tie, 2014b). Consider a scalar system

$$\dot{y} = -\alpha y^{\frac{m}{n}} - \beta y^{\frac{p}{q}}, \quad y(0) = y_0 \quad (3)$$

where $\alpha > 0$, $\beta > 0$, m, n, p, q are positive **odd** integers satisfying $m > n$ and $p < q$. The equilibrium of (3) is globally fixed-time stable with settling time T bounded by

$$T < T_{\max} := \frac{1}{\alpha} \frac{n}{m-n} + \frac{1}{\beta} \frac{q}{q-p}. \quad (4)$$

If $\varepsilon \triangleq [q(m-n)]/[n(q-p)] \leq 1$, a less conservative estimation of the settling time can be obtained instead as

$$T < T_{\max} := \frac{q}{q-p} \left(\frac{1}{\sqrt{\alpha\beta}} \tan^{-1} \sqrt{\frac{\alpha}{\beta}} + \frac{1}{\alpha\varepsilon} \right). \quad (5)$$

2.3. Problem formulation

Consider a group of N continuous-time agents as

$$\dot{x}_i(t) = v_i(t), \quad \dot{v}_i(t) = u_i(t) \quad (6)$$

where $x_i \in \mathbb{R}$, $v_i \in \mathbb{R}$ and $u_i \in \mathbb{R}$ denote the position, velocity and input (protocol) of the i th agent respectively, and $i \in \mathcal{I}_N \triangleq \{1, 2, \dots, N\}$.

Let the consensus reference state evolution be

$$\dot{x}_0(t) = v_0(t), \quad \dot{v}_0(t) = u_0(t) \quad (7)$$

where $x_0 \in \mathbb{R}$, $v_0 \in \mathbb{R}$ and $u_0 \in \mathbb{R}$ denote the position, velocity and input of the reference system. We introduce a virtual leader π_0 with the states $\xi_0 = [x_0, v_0]^T$ for the multi-agent system in (6). Note, however, that ξ_0 is available not to all agents but to only a portion of agents. Here, define a nonnegative diagonal matrix $B = \text{diag}\{b_1, b_2, \dots, b_N\}$ to indicate the accessibility of ξ_0 by the agents, where $b_i = 1$ if ξ_0 is accessible by the i th agent, and $b_i = 0$ otherwise. The directed graph incorporating π_0 into \mathcal{G} is denoted by \mathcal{G}^e .

Assumption 3. \mathcal{G}^e has a spanning tree with π_0 being its root vertex, i.e., $B \neq 0$.

Assumption 4. The input $u_0(t)$ of the leader is unavailable to the group members, but its upper-bound u_0^M is accessible by its neighboring agents.

With a given protocol u_i , the closed-loop system in (6) is said to reach or achieve *fixed-time consensus tracking*, if, for $\forall \xi_i(0)$ and $\forall i, j \in \mathcal{I}_N$ there exists a constant $T_{\max} > 0$ such that the *settling time* $T < T_{\max}$ and

$$\begin{cases} \lim_{t \rightarrow T} |\xi_i(t) - \xi_0(t)| \rightarrow 0 \\ \xi_i(t) = \xi_0(t), \quad \forall t \geq T \end{cases} \quad (8)$$

where $\xi_i \triangleq [x_i, v_i]^T$, $|\cdot|$ is defined componentwise.

3. Nonsingular fixed-time control

To clarify the core idea, we first consider the fixed-time control of a single system defined by

$$\dot{z}_1(t) = z_2(t), \quad \dot{z}_2(t) = u(t) \quad (9)$$

where $z = [z_1, z_2]^T \in \mathbb{R}^2$ denotes the system state vector, $u \in \mathbb{R}$ the control input.

Let m_k, n_k, p_k, q_k be positive odd integers satisfying $m_k > n_k$, $p_1 < q_1 < 2p_1$, $p_2 < q_2$ and $m_1/n_1 - p_1/q_1 > 1$, and α_k, β_k be positive constants, where $k = 1, 2$. Define a \mathcal{C}^1 function $\mu_\tau(\cdot) : [0, +\infty) \rightarrow [0, 1]$ as

$$\mu_\tau(x) = \begin{cases} \sin\left(\frac{\pi}{2} \cdot \frac{x}{\tau}\right) & \text{if } x \leq \tau \\ 1 & \text{otherwise} \end{cases} \quad (10)$$

where τ is a positive constant.

To circumvent the singularity problem (Feng et al., 2013), a new sliding surface is proposed:

$$s = z_1 + [\kappa(z_1) \cdot z_2]^{\frac{q_1}{p_1}} \quad (11)$$

where $\kappa(\cdot) : \mathbb{R} \rightarrow \mathbb{R}^+$ denotes a scalar positive function, given by

$$\kappa(x) = \frac{1}{\alpha_1 x^{m_1/n_1 - p_1/q_1} + \beta_1}. \quad (12)$$

In the sequel, the parameter in $\kappa(\cdot)$ will be omitted for simplicity after its first definition. It can be straightforwardly verified that $s = 0$ implies

$$z_2 = -\alpha_1 z_1^{\frac{m_1}{n_1}} - \beta_1 z_1^{\frac{p_1}{q_1}}.$$

A new fixed-time nonlinear control law is defined as

$$\begin{aligned} u = & \frac{1}{\kappa} \left[\alpha_1 \left(\frac{m_1}{n_1} - \frac{p_1}{q_1} \right) z_1^{\frac{m_1}{n_1} - \frac{p_1}{q_1} - 1} (\kappa z_2)^2 - \frac{p_1}{q_1} \kappa^{1 - \frac{q_1}{p_1}} z_2^{2 - \frac{q_1}{p_1}} \right] \\ & - \frac{p_1}{q_1} \kappa^{-\frac{q_1}{p_1}} \cdot \mu_\tau \left(z_2^{\frac{q_1}{p_1} - 1} \right) \cdot z_2^{1 - \frac{q_1}{p_1}} \left(\alpha_2 s^{\frac{m_2}{n_2}} + \beta_2 s^{\frac{p_2}{q_2}} \right) \end{aligned} \quad (13)$$

where $\mu_\tau(\cdot)$ is defined in (10) with $x = z_2^{q_1/p_1 - 1}$. The fact $\sin \omega x \leq x$, $\forall \omega > 0$ ensures the input (13) is always well-defined. The following theorem presents the novel *fixed-time* stability property (Polyakov, 2012).

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