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Conditions for transmission path analysis in energy distribution models

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ABSTRACT

In this work, we explore under which conditions transmission path analysis (TPA) developed for statistical energy analysis (SEA) can be applied to the less restrictive energy distribution (ED) models. It is shown that TPA can be extended without problems to proper-SEA systems whereas the situation is not so clear for quasi-SEA systems. In the general case, it has been found that a TPA can always be performed on an ED model if its inverse influence energy coefficient (EIC) matrix turns to have negative off-diagonal entries. If this condition is satisfied, it can be shown that the inverse EIC matrix automatically becomes an *M*-matrix. An ED graph can then be defined for it and use can be made of graph theory ranking path algorithms, previously developed for SEA systems, to classify dominant paths in ED models. A small mechanical system consisting of connected plates has been used to illustrate some of the exposed theoretical results.

1. Introduction

Transmission path analysis (TPA) in statistical energy analysis (SEA) relies on Craik's definition of energy transmission path [1]. TPA is usually carried out for vibroacoustics remedial purposes as it provides information on energy flow paths from a source subsystem, where external energy is being input, to a target subsystem where energy is to be reduced [2]. Performing a TPA basically makes sense whenever the energy transmission is dominated by a limited set of paths. However, computing and ranking transmission paths in an efficient way are not a straightforward task for complex SEA models. A procedure to do so was recently proposed in [3,4], taking advantage of a link established between SEA and graph theory [5] (see also [6] for further possibilities exploiting this connection).

Given that a mechanical system and its inputs have to satisfy some rather restrictive hypotheses in order to be modeled by SEA, namely modal energy equipartition, diffuse field, weak coupling, rain-on-the-roof excitation (see e.g., [7–9] for details on the connections among these conditions), it is the main goal of this work to determine under which circumstances a SEA-like TPA could be extended to the more general energy distribution (ED) models. In an ED model, the mechanical system also becomes split into a set of subsystems, and the energy at any of them when submitting some part of the system to a broadband excitation is characterized by means of the so-called energy influence coefficients (EICs) ([10], see also [11,12]). The EICs can be computed either from theoretical modal developments [10], numerical approaches using the finite element method [11–13], or by resorting to experimental procedures relying on the power injection method [14–17]. They are not submitted to the restrictive conditions demanded to SEA coupling loss factors. Actually, a question of

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interest is precisely that of determining under which circumstances the EICs fulfill the appropriate conditions that let one identifying the ED matrix with an SEA loss factor matrix. Initial numerical studies on that line were carried out in [18], whereas the definition of quasi and proper seA systems was proposed in [19,20]. In the present work we will explore whether it is possible or not to apply a TPA to quasi and proper seA models, as well as to more general ED systems. In the propitious cases, ED graphs could be built to compute and rank transmission paths.

In this sense, it is worthwhile mentioning that it has very recently been proved that such type of TPA analysis can be actually performed for SmEdA (Statistical modal Energy distribution Analysis) models [21]. SmEdA circumvents some of the SEA requirements by establishing power balance equations between modes in subsystems rather than between subsystems themselves. It thus avoids the necessity of modal energy equipartition and allows one to deal with situations with low modal overlap, complex heterogeneous subsystems and point-like external excitations [22–24]. The price to be paid is that of larger matrix systems than in SEA, so that graph algorithms become very useful to identify which modes govern resonant and non-resonant transmission in SmEdA models.

The paper is organized as follows. In Section 2 we present a brief review of ED models and introduce the EIC matrix as well as its inverse. In Section 3 we give a possible definition for transmission paths in ED models and justify it in terms of the series expansion of the subsystem energy vector. Then, we determine under which conditions this expansion will converge making ED TPA possible in some cases. A small ED consisting of six steel plates is used in Section 4 to show how a TPA can be performed for a system that does not admit SEA modeling. Finally, the conclusions close the paper in Section 5.

2. Energy distribution models

In an ED model of a linear mechanical system, the relation between the external input power $\boldsymbol{P}_{in}$ into the constituent subsystems and their energies $\boldsymbol{E}$ takes place through an energy influence coefficient (EIC) matrix $\boldsymbol{A}$, so that (see [19])

$$\boldsymbol{E} = \boldsymbol{A}\boldsymbol{P}_{in}. \tag{1}$$

The external excitations in (1) are assumed to be random and stationary, and uncorrelated between subsystems. Moreover, the input power and subsystem energies are time and frequency averaged over the considered frequency band. The EIC between two subsystems i and j, A_{ij}, consequently represents the time and frequency averaged energy at i, resulting from a unit input power at subsystem j. The EIC matrix $\boldsymbol{A}$ can be directly obtained from experiments, or from numerical or analytical modeling.

Let us next consider the inverse of the EIC matrix $\boldsymbol{X} := \boldsymbol{A}^{-1}$. It follows that

$$\boldsymbol{X}\boldsymbol{E} = \boldsymbol{P}_{in}, \tag{2}$$

where the i-th row expresses power balance for the i-th subsystem. Eq. (2) reminds of the structure of a SEA matrix system $\boldsymbol{H}\boldsymbol{E} = \boldsymbol{P}_{in}$ ($\boldsymbol{H}$ standing for ω times the SEA matrix of loss factors, ω being the radial frequency). However, $\boldsymbol{X}$ is much more general than $\boldsymbol{H}$, and does not need to fulfill the SEA hypotheses.

The question of which conditions the inverse EIC matrix $\boldsymbol{X}$ should satisfy to become an SEA loss factor matrix has been addressed by various authors, starting with the numerical analysis in [18]. In [19,20], a distinction was made between the so-called *quasi-SEA* and *proper-SEA* matrices. $\boldsymbol{X}$ was said to be a quasi-SEA matrix if it satisfies the two necessary conditions of energy balance and consistency. The former refers in [19,20] to the fact that the j-th column of $\boldsymbol{X}$ must sum to $\omega\eta_j$, with η_j denoting the damping loss factor of subsystem j. The second condition concerns an analogous for matrix $\boldsymbol{X}$ off-diagonal elements of the well-known consistency relation in SEA $n_i\eta_{ij} = n_j\eta_{ji}$ (n_i stands for the modal density of subsystem i and η_{ij} for the coupling loss factor between i to j). The consistency relation is a direct consequence of the coupling power proportionality (CPP) hypothesis (see e.g. [25]). If in addition to these conditions, all indirect coupling loss factors between subsystems are zero, then $\boldsymbol{X}$ was termed a proper-SEA matrix. It is to be noted that a proper-SEA matrix is not yet a SEA matrix, given that X_{ij} may not comply with some of the ideal properties a SEA coupling loss factor should have. For instance, X_{ij} could depend on the damping loss factor which is not the case for η_{ij}. Although matrix $\boldsymbol{X}$ always satisfies energy balance, in the most general case it does not even have to be SEA-like because fulfillment of the CPP condition is not guaranteed (see [20]).

As commented in the Introduction, the aim of this work is to explore under which conditions transmission path analysis becomes possible in an ED model, so that previously developed graph algorithms [3] can be applied to it. It will be shown that this depends on some properties of $\boldsymbol{X}$, though no direct one to one correspondence has been found between them and those involved in the above definitions of SEA-like matrices. We will see that transmission path analysis makes sense for proper-SEA matrices as well as for any other $\boldsymbol{X}$ matrix that can be shown to be an M-matrix (similarly, for any EIC matrix $\boldsymbol{A}$ that can be shown to be an *inverse* M-matrix).

3. Transmission path analysis in energy distribution models

3.1. Definition of transmission paths

The definition of transmission paths in SEA systems was introduced by Craik in [1] (see also [2]). An energy path between two arbitrary subsystems in a SEA system is made of the concatenation of first order paths between adjacent subsystems. Let us denote a first order path from subsystem i to j by p^1_{ij}. Its weight is given by $w(p^1_{ij}) = \eta_{ij}/\eta_j$ (η_j standing for the total loss