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Identification of the direction and value of the wave length of each mode for a rotating tire using the phase difference method

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ABSTRACT

Natural frequencies, mode shapes and modal damping values are the most important parameters to describe the noise and vibration behavior of a mechanical system. For rotating machinery, however, the directivity of the propagation wave and the wave length of each mode should also be taken into account. Generally, the information on directivity and wave length is obtained on the basis of the mode shape result, which is estimated from several measurements measured at different locations. In this research, the accurate directivity and wave length results will be observed by calculating the phase difference at two different locations. The limitation of the proposed method, which arises from the difference between the assumed ring model and the real tire, will be explained, and a method to address the limitation is introduced. The proposed method is verified by applying it to experimental measurements, and a brief explanation of the obtained results is provided.

1. Introduction

As the number of vehicles has increased significantly over the past few decades, urban inhabitants have been exposed to uncomfortable noise, which arises mainly due to the movement of vehicles while driving [1]. In particular, the noise and vibration that are transmitted by the interaction of the tires of a vehicle and the road surface are the main causes of the harshness of the noise to urban inhabitants [2]. Therefore, to reduce the noise and vibration during driving, research on the dynamic characteristics of a tire as well as of the vehicle itself has been actively pursued in recent years [3–5]. Typically, modal analysis is used to determine the dynamic characteristics of a tire. For the stationary case, modal analysis can be performed based on the frequency response function, which is obtained by the measured forces and responses [6]; however, for the rolling case, operational modal analysis should be adopted, which uses only responses because of the restriction of the experimental setup to measure the applied forces [3]. The rolling tire experiences bifurcation phenomena due to the Coriolis Effect, in which the natural frequency of the each mode is separated into two distinct frequencies [7,8]; to identify this phenomenon through experimentation, the directivity of the propagating wave must be determined through the mode

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shapes. Therefore, responses at multiple locations are required to study the dynamic properties of a rotating tire; however, this requirement can result in high expenditure, and difficulties can arise in the development of the experimental configuration. In this study, a technique is proposed that uses a lower amount of measurement data to reduce the difficulty of the experimental configuration; this technique is used to estimate the modal parameters, which enables the prediction, in particular, of the wave length and the directivity of the each mode from the phase differences of the measurements.

This paper is organized as follows. In Section 2, the theoretical background of the proposed technique is introduced, and in Section 3, the experimental configuration and results are studied to verify the proposed technique. Finally, conclusions are summarized in Section 4. More specifically, in Section 2.1, the ring model that is used in this study is introduced and the assumptions for this model are included. In Section 2.2 the detailed description for the proposed technique is introduced by explaining the analytic model. In addition, the difference between a real rolling tire and the introduced model is discussed, and the possibility of the proposed technique to overcome the difference is described.

2. Theory

2.1. Ring model

In previous studies, a rotating flexible ring was used to analyze a rotating tire [7,9,10]; here, a flexible ring represents the in-plane vibration of the belt and tread layer of a rotating tire, and two stiffness values in the radial and circumferential directions, which are connected on an elastic foundation, represent the tire sidewall. The vibrational properties in the torsional direction are difficult to estimate using a flexible ring model; however, because these vibrations generally occur above 300 Hz, a flexible ring model is sufficient to analyze the vibrational properties of the tire below this frequency region [3]. The equations of motion for a flexible rotating ring in cylindrical coordinate are [7]

$$\frac{EI}{R^4}\left(\frac{\partial^4 u_r}{\partial\theta^4}-\frac{\partial^3 u_\theta}{\partial\theta^3}\right)+\frac{EA}{R^2}\left(\frac{\partial u_\theta}{\partial\theta}+u_r\right)+\frac{\sigma_\theta^0 A}{R^2}\left(\frac{\partial u_\theta}{\partial\theta}-\frac{\partial^2 u_r}{\partial\theta^2}+R\right)$$
$$+k_r u_r-p_0 h\left(1+\frac{1}{R}\left(\frac{\partial u_\theta}{\partial\theta}+u_r\right)\right)+\rho A\left(\ddot{u}_r-2\Omega\dot{u}_\theta-(u_r+R)\Omega^2\right)=q_r \quad (1-1)$$

$$\frac{EI}{R^4}\left(\frac{\partial^3 u_r}{\partial\theta^3}-\frac{\partial^2 u_\theta}{\partial\theta^2}\right)-\frac{EA}{R^2}\left(\frac{\partial u_r}{\partial\theta}+\frac{\partial^2 u_\theta}{\partial\theta^2}\right)+\frac{\sigma_\theta^0 A}{R^2}\left(u_\theta-\frac{\partial u_r}{\partial\theta}\right)+k_\theta u_\theta$$
$$+\rho A\left(\ddot{u}_\theta+2\Omega\dot{u}_r-\Omega^2 u_\theta\right)=q_\theta \quad (1-2)$$

$$u_r(\theta,t)=U_r e^{j(n\theta+\omega t)},\ u_\theta(\theta,t)=U_\theta e^{j(n\theta+\omega t)} \quad (1-3)$$

Eq. (1) is based on four assumptions [3,7]. First, transverse shear deflections ($\varepsilon_{\theta r}$) are neglected. Second, all displacements in the z-direction are considered to be constant over the belt width; moreover, stresses and deformations in this direction are assumed to be zero ($\varepsilon_{zz}=0$, $\varepsilon_{\theta z}=0$). Third, the stress acting in the normal surface to the normal direction is neglected. Finally, the Love simplification is applied. In Eq. (1), u_r and u_θ represent the displacements in the radial and circumferential directions, respectively, and the basic form of these displacements are represented in Eqs. (1-3). Here, n and ω represent the values of wave length and wave frequency, respectively, that is, the two directional displacements are composed of a function of the spatial and time variables. Young's modulus, moment of inertia, radius and width of the ring are represented by E, I, R and h, respectively. In addition, A represents the cross-sectional area (b (width) $\times h$) and p_0 and Ω are the inner pressure and the rotational speed, respectively. Two directional stiffness and distributed load per unit area are represented by k_r, k_θ, q_r and q_θ, respectively. Here, the subscripts r and θ indicate the radial and circumferential directional components, respectively. The pretension force $\sigma_\theta^0 A$ is represented as

$$\sigma_\theta^0 A=\left(p_0 bR+\rho AR^2\Omega\right) \quad (2)$$

The first and second terms on the right hand side in Eq. (2) are the induced forces due to the inflation pressure and the centrifugal force, respectively. The equation of motion in Eq. (1) is represented in local (Lagrangian) coordinates, but it should be transformed into global (Eulerian) coordinates by the application of Reynolds' theorem because laser Doppler vibrometry (non-contact sensor) is used in this research to measure the responses [8]. The mathematical expression of the theorem is given below:

$$\frac{D}{Dt}=\frac{\partial}{\partial t}+\Omega\frac{\partial}{\partial\theta} \quad (3)$$

The left hand side of Eq. (3) represents the local coordinates (Lagrangian), and the first term on the right hand side represents the global coordinates (Eulerian). If this equation is applied to Eq. (1), then the resultant can be represented by matrix form

$$M\ddot{\mathbf{u}}(\theta,t)+\mathbf{C}\dot{\mathbf{u}}(\theta,t)+\mathbf{K}\mathbf{u}(\theta,t)=\mathbf{q}(\theta,t) \quad (4)$$