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Equivalent Dynamic Stiffness Mapping technique for identifying nonlinear structural elements from frequency response functions

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ABSTRACT

A simple and general Equivalent Dynamic Stiffness Mapping technique is proposed for identifying the parameters or the mathematical model of a nonlinear structural element with steady-state primary harmonic frequency response functions (FRFs). The Equivalent Dynamic Stiffness is defined as the complex ratio between the internal force and the displacement response of unknown element. Obtained with the test data of responses' frequencies and amplitudes, the real and imaginary part of Equivalent Dynamic Stiffness are plotted as discrete points in a three dimensional space over the displacement amplitude and the frequency, which are called the real and the imaginary Equivalent Dynamic Stiffness map, respectively. These points will form a repeatable surface as the Equivalent Dynamic stiffness is only a function of the corresponding data as derived in the paper. The mathematical model of the unknown element can then be obtained by surface-fitting these points with special functions selected by priori knowledge of the nonlinear type or with ordinary polynomials if the type of nonlinearity is not pre-known. An important merit of this technique is its capability of dealing with strong nonlinearities owning complicated frequency response behaviors such as jumps and breaks in resonance curves. In addition, this technique could also greatly simplify the test procedure. Besides there is no need to pre-identify the underlying linear parameters, the method uses the measured data of excitation forces and responses without requiring a strict control of the excitation force during the test. The proposed technique is demonstrated and validated with four classical single-degree-of-freedom (SDOF) numerical examples and one experimental example. An application of this technique for identification of nonlinearity from multiple-degree-of-freedom (MDOF) systems is also illustrated.

1. Introduction

The linear assumption of a structure may be violated due to the existing of nonlinear elements, or nonlinear boundaries, which possess significant amplitude-dependent or frequency-dependent characteristics that can no longer be neglected. The fundamental harmonic frequency response function (FRF) might be distorted, for example, the resonance peak shifts and the amplitude varies with different load levels, and the jump phenomenon may even occur. In order to accurately

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describe, characterize and predict the structural responses, a substantial large body of literature has been devoted to the detection and identification of nonlinearity in structures. A text book [1] written by Worden and Tomlinson and a comprehensive review paper [2] published by Kershen et al. describe most of the existing theories and techniques aimed to identify nonlinearities from structural tests. Up till now, most of these methods are applicable to numerical examples, only a scarce number of them are still efficient and easy to implement when dealing with real test results of nonlinear industrial structures. Such methods should well reflect the nonlinear effects and be not too mathematically complicated. More importantly, a method suitable for practical application should be compatible with the existing test procedures employed by the engineer community.

Some of the most promising methods will be briefly reviewed in this paper. The restoring force surface method [3] and another independently developed Force-State Mapping method [4] are the early developed and most widely used time domain identification methods. They are essentially based on the same nonparametric technique that plotting and fitting the restoring force surface over the state, i.e. velocity and displacement, of virtually any type of nonlinear element. This technique needs to simultaneously measure the time series of the restoring force and the state, while the excitation could be arbitrary (for example, random or swept-sine [5] inputs) during the identification process. Due to its mathematical simplicity and nonparametric merit, this time domain mapping method has been widely applied to identification of joints [6], piecewise linear beam [7], etc. Developments of this method have been made to identify nonlinearity from MDOF systems [8], locate the nonlinear elements [5], identify nonlinearity without input measurements [9], or perform an advanced test for aerospace structures [10]. An interesting extension was made by Kim and Park [11] to apply this method to determine the relationship between the joint forces and the joint coordinates with joint responses in the frequency domain, which are less sensitive to noise.

Identification methods based on measured frequency response functions (FRFs) have also attracted a lot of attention over the last two decades since it is a common practice to obtain FRFs of real-life structures in the industrial test procedures. The linearity plot method [10,12,13], in which the resonance frequencies or damping ratios are plotted as functions of different excitation levels, is arguably the most convenient method to implement on large scale structures. Nevertheless, this method could only be used to detect and characterize the nonlinearities instead of quantify them. Another rigorous method is based on the concept of Nonlinear Normal Modes (NNMs) introduced by Rosenberg [14,15] aiming to extract the nonlinear modal parameters from FRFs. It has been widely implemented to the identification of nonlinear normal modes of an aircraft landing gear based on the frequency curve fitting [16], a benchmark problem of a cantilever beam with a thin beam [17], and nonlinear boundary effects due to micro-slip/slap [18]. Literature reviews of this method could be found in Refs [2,19].

A critical procedure during the implementation of these frequency domain methods is the way to obtain nonlinear FRFs of a real structure. Recently, the constant response level testing method has been considered by many researchers. During the test, dedicated excitation signals are employed to control the response amplitude as a constant using a feedback control system [20,21]. In this way, the FRFs of the nonlinear structure are actually linearized at this response level to avoid nonlinear distortion, and thus the conventional linear analysis tools could be used to extract modal data from the measured FRFs. This method has been successfully applied to identify bolted flange of an aircraft engine [20], a typical STM satellite structure [20] and a cantilever beam with hardening stiffness nonlinearity [21]. Nevertheless, it is time consuming and difficult to maintain a constant response during the test. Improvements have been made by Carrella and Ewins [22] to introduce a different linearization approach by extracting modal data from a complex pair of points, which has the same amplitude response in the FRF curve. This method may fail when jump occurs and thus leads to poor results. Experimental data in this research [22] include the Nastran tower and a complex helicopter structure with weak nonlinearities. Nevertheless, these constant amplitude linearization methods have their own drawbacks. For instance, they are restricted to amplitude-dependent nonlinearities and usually only one nonlinear element could be considered in the structure, since it is neither likely to maintain nor find enough constant responses of several nonlinear elements during a test. This constant amplitude requirement is released in the second method introduced in Ref [21] by bringing in the Describing Function and the Sherman–Morison formula that was first proposed by Özer et al. in Ref. [23]. The obtained describing function values are plotted over the displacement response amplitude, which do not need to be constant, and the curve fitted to the function data is used to describe the nonlinearity. In this way, this method is not restricted to the requirement of constant amplitude. It should be noted that the nonparametric identification process in this method is still limited to the amplitude-dependent stiffness and/or damping [21,23,25]. The frequency-dependent nonlinearities, which are common in nonlinear damped structures, however, has to be curve fitted only when the type of nonlinearity is pre-known and its underlying linear system is pre-identified [24], during which the responses might easily be contaminated by the noise as the underlying linear parameters are often estimated at very low forcing levels [21,24,25]. Moreover, in some special cases such as the dry friction, low forcing level inputs will also introduce significant nonlinear responses [25,26].

As have been reviewed, the parametric and nonparametric identification methods based on the FRFs still require extension, especially for damping nonlinearities that might depend on both amplitude and frequency. The intent of this paper is to develop a simple and generalized identification procedure based on the measured FRFs, which could be obtained under stepped-sine excitations. The Equivalent Dynamic Stiffness, which is defined as the linear and/or nonlinear ratio between the complex force and displacement in this paper, are first calculated from the FRFs without pre-identifying the underlying linear structure. Next, the real part and the imaginary part of the Equivalent Dynamic Stiffness are plotted as discrete points in a three-dimensional space over displacement amplitude and frequency, which are called real and imaginary Equivalent Dynamic Stiffness map in this paper, respectively. By surface-fitting these points with reasonable basis

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