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# Techniques for estimating the unknown functions of incomplete experimental spectral and correlation response matrices

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ABSTRACT

In this paper, we propose analytical and numerical straightforward approximate methods to estimate the unknown terms of incomplete spectral or correlation matrices, when the cross-spectra or cross-correlations available from multiple measurements do not cover all pairs of transducer locations. The proposed techniques may be applied whenever the available data includes the auto-spectra at all measurement locations, as well as selected cross-spectra which implicates all measurement locations. The suggested methods can also be used for checking the consistency between the spectral or correlation functions pertaining to measurement matrices, in cases of suspicious data. After presenting the proposed spectral estimation formulations, we discuss their merits and limitations. Then we illustrate their use on a realistic simulation of a multi-supported tube subjected to turbulence excitation from cross-flow. Finally, we show the effectiveness of the proposed techniques by extracting the modal responses of the simulated flow-excited tube, using the SOBI (Second Order Blind Identification) method, from an incomplete response matrix [1].



## 1. Introduction

Given a set of $R$ response measurement locations in vibratory or acoustical experiments, several system identification techniques make use of a complete set of $R \times R$ cross-spectral response estimates $S_{ij}(f)$, with $i,j = 1,2,\ldots,R$. Examples of such techniques include the SVD decomposition of cross-spectral measurement matrices $\mathbf{S}_{YY}(f)$, as a preliminary step for modal identification purposes:

$$\mathbf{S}_{YY}(f) = \mathbf{U}(f)\mathbf{\Sigma}(f)\mathbf{U}(f)^H \tag{1}$$

here, at each frequency, the columns of matrix $\mathbf{U}(f)$ and its Hermitian transpose $\mathbf{U}(f)^H$ contain the singular vectors of $\mathbf{S}_{YY}(f)$ and the terms of the diagonal matrix $\mathbf{\Sigma}(f)$ are the corresponding singular values. Such techniques, introduced by Brincker et al. [1] and often designated FDD (Frequency Domain Decomposition), recently became widely used for the purpose of the

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so-called operational modal identification, when dealing with large structures subjected to unmeasured random excitations, such as bridges and towers under the excitation of wind or traffic, see for instance [2–4].

Obviously, because of reciprocity, one has $S_{ji}(f) = S^{*}_{ij}(f)$ (where the starred function refers to complex conjugate), meaning that only the $(0.5(R-1)+1)R$ cross-spectra of the upper triangular terms must be measured, instead of the full $R^2$ terms of the spectral matrix. Even so, this may represent a significant amount of measurements to be performed, involving either many transducers or many transducer location changes, and possibly the need for very long signal cables, as illustrated in Fig. 1. All these features are certainly inconvenient and often non-feasible. Therefore, we believe that procedures which would enable the “filling” of incompletely measured spectral matrices might be very convenient and useful in many instances. In the following we will show that, provided some essential assumptions apply to the analyzed system and data, simple approximate methods may be devised for such a purpose.

Three straightforward methods for estimating the unknown functions – two analytical and a numerical one – are suggested and discussed in this paper, which can be applied in many situations of practical interest. In the two proposed analytical reconstruction approaches, the assumed minimum of available experimental data includes the $R$ auto-spectra $S_{ii}(f)$ from all the measurement locations, as well as $R-1$ cross-spectra involving all transducer locations. The numerical reconstruction approach may easily deal with other configurations of known measurements. A specific set of available cross-spectra, which will be of particular interest here because it fits the experimental constraints of our work, involves measurements between consecutive transducers: $S_{12}(f),\ S_{23}(f),\ \ldots,\ S_{i(i+1)}(f),\ \ldots,\ S_{(R-2)(R-1)}(f),\ S_{(R-1)R}(f)$. The simple estimation techniques proposed in the following apply to spectral matrices $\mathbf{S}_{YY}(f)$, manipulations being easier in the frequency domain. Nevertheless results also apply to correlation matrices $\mathbf{R}_{YY}(\tau)$, as the former are related to the later through the inverse Fourier Transform $R_{ij}(\tau) = \boldsymbol{F}^{-1}(S_{ij}(f))$.

Beyond reducing the number of response cross-measurements from about $\boldsymbol{O}(N^2)$ to $\boldsymbol{O}(N)$, the suggested estimation methods may also be used for checking the consistency between the spectral or correlation functions pertaining to measurement matrices, in cases of suspicious experimental data. After presenting the proposed approximate formulations and spectral estimation techniques, we discuss their merits and limitations. Then we illustrate their use on realistic simulations of a multi-supported tube subjected to turbulence excitation from cross-flow. Finally, we show the effectiveness of the proposed techniques by extracting the modal responses of the simulated flow-excited tube, using the SOBI (Second Order Blind Identification) method, from incomplete response matrices.

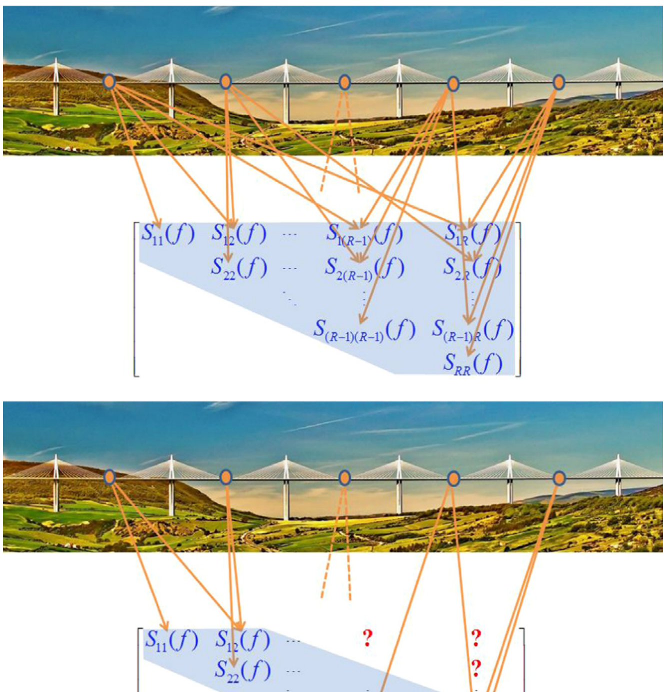

**Fig. 1.** Possible collecting strategies of the response data for performing system identification under unmeasured excitations: Extensive measurement of the complete spectral matrix (upper plot); Measurement of selected auto- and cross-spectra (lower plot).