Mechanical Systems and Signal Processing ∎ (∎∎∎∎) ∎∎∎–∎∎∎

Contents lists available at ScienceDirect

Mechanical Systems and Signal Processing

journal homepage: www.elsevier.com/locate/ymssp

Static output feedback for partial eigenstructure assignment of undamped vibration systems

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ARTICLE INFO

Article history:
Received 26 March 2015
Received in revised form
7 June 2015
Accepted 8 July 2015

Keywords:
Partial eigenstructure assignment
Static output feedback
Undamped vibration system
Acceleration and displacement feedback

ABSTRACT

A novel method for partial eigenstructure assignment of undamped vibration systems using acceleration and displacement output feedback is presented in this paper. It is based on modifications of mass and stiffness that preserve partial eigenstructure. A numerical algorithm for determining the required control gain matrices of acceleration and displacement output feedback, which assign the desired eigenstructure, is developed. This algorithm is easy to implement, and works directly on the second-order system model. More importantly, the algorithm allows the output matrix and the input matrix to be specified beforehand and also leads naturally to a small norm solution of the gain matrices. Finally, some numerical results are presented to demonstrate the effectiveness and accuracy of the proposed algorithm.

1. Introduction

Active vibration control techniques of engineering structures have been extensively studied during the past three decades. The dynamic response of a vibrating system can be altered by changing the system's natural frequencies and mode shapes, namely, its modal characteristics, which is also referred to as the eigenstructure (i.e. the eigenvalues and eigenvectors). Thus the eigenvalue or eigenstructure reallocation or assignment is a common control strategy in active vibration suppression.

Eigenvalue assignment and eigenstructure assignment working directly on second-order dynamic system models has attracted much attention over the last 10 years, partly because of the demands in general control and vibration control applications in engineering, and partly because of the advantage of those peculiar properties afforded by the second-order system models. Another issue being taken into account in the area of research is that, in real applications, it is needed to change only a few undesirable eigenvalues or undesirable part of eigenstructure which are purposefully assigned to desired values, and it is desirable to keep all other eigenpairs unchanged. This problem is called *partial eigenvalue or eigenstructure assignment*. Some major effort can be seen from the literature to tackle this problem, for example, in [1–14] on damped and undamped second-order vibration systems.

All these above approaches solved the problem by *full-state feedback*, but in most practical situations the full states are not directly available. From a practical standpoint, a more attractive procedure would be one which is based upon feeding back only the measured variables, i.e., *static output feedback* (SOF). For eigenvalue or eigenstructure assignment of first-order state-space system models via SOF, a great deal of research exists, and numerical algorithms and some readily verifiable necessary or sufficient conditions for determining solvability have been proposed. Many results, however, are mainly

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http://dx.doi.org/10.1016/j.ymssp.2015.07.012
0888-3270/

theoretical in nature and there are no good numerical algorithms available in many cases when a specific system is known to be solvable. It is believed that the solution techniques that work well on small-sized systems may be doomed as the system size increases [15]. Someone suggests that every effort should be made to exploit the particular structure of a given SOF problem. The starting points for further information about SOF are the survey papers [15,16], as well as the more recent papers [17–19]. As for those working directly on second-order system models via SOF, few results can be seen from the literature. Lin and Wang proposed a solution to the partial eigenvalue assignment problem for the second-order damped vibration system models by SOF [20]. They considered the elements of the output matrix and the input matrix as design variables as well, and explained the research problem for this setting in [20] as follows: *For the usual partial eigenvalue assignment problem by output feedback, the input and output matrices are in general fixed. However, it seems very difficult to relocate unwanted eigenvalues to desired values while keeping all wanted eigenstructure unchanged with fixing input and output matrices. To our knowledge, there is no result in this direction.* In addition, they set the input matrix to be the transpose of the output matrix, namely, the collocated actuator and sensor configuration.

In this paper we attack the partial eigenstructure assignment problem by SOF for second-order undamped vibration system models. The main contribution of this paper consists of the following: (1) The input matrix and the output matrix here can be prescribed and chosen in a simple form, and the collocated actuator and sensor configuration is not necessary. Two measured variables, the acceleration and displacement, are used and correspondingly there are two output matrices, respectively. (2) The proposed algorithm only needs those few eigenpairs to be assigned and the analytical mass and stiffness matrices of the original vibration system, and also leads naturally to a small norm solution of the output feedback gain matrices.

The work here is based on our recent article [14] where we obtained a partial eigenstructure modification formulation. In [14] a necessary and sufficient condition was proposed for the incremental mass and stiffness matrices that modify some eigenvalues or eigenpairs while keeping other eigenpairs unchanged, and an efficient numerical algorithm was suggested for partial eigenstructure assignment of undamped vibration systems using acceleration and displacement state feedback. In what follows, the partial eigenstructure modification formulation is presented, and the problem involved, some notations and assumptions are described in Section 2. A partial eigenstructure assignment algorithm is proposed to determine the acceleration and displacement output feedback gain matrices in Section 3. In Section 4, some numerical results are provided to demonstrate the effectiveness of the proposed method.

2. The problem description

2.1. A partial eigenstructure modification formulation

Consider an n-degree-of-freedom undamped vibration system that is modelled by the following set of second-order ordinary differential equations:

$$M_0\ddot{q}(t)+K_0q(t)=f(t) \tag{1}$$

where $q(t)\in R^n$ is the displacement vector, $f(t)\in R^n$ is the vector of external forces, and M_0, and $K_0\in R^{n\times n}$ are constant mass and stiffness matrices, respectively. In general, M_0 is symmetric and positive definite, and K_0 is symmetric and positive semi-definite, i.e. $M_0=M_0^T>0$, $K_0=K_0^T\geq 0$.

It is well known that if $q(t)=xe^{j\omega t}$ is a fundamental solution of (1), then the natural frequency ω and the mode shape vector x must satisfy the following generalized eigenvalue equation:

$$(K_0-\lambda_i M_0)x_i=0,\quad i=1,2,\dots,n \tag{2}$$

where the ith eigenvalue $\lambda_i=\omega_i^2$ is the square of the ith natural frequency ω_i, and x_i is the corresponding ith eigenvector. Eq. (2) can be written in a compact representation as follows:

$$K_0X=M_0X\Lambda \tag{3}$$

where $\Lambda=\mathrm{diag}(\lambda_1,\lambda_2,\dots,\lambda_n)$ and $X=(x_1,x_2,\dots,x_n)$ make up the complete eigenstructure of the system (1), and X satisfies the mass-normalised condition $X^TM_0X=I_n$.

Suppose that the system described by (1) is modified by the incremental mass and stiffness matrices $\Delta M\in R^{n\times n}$ and $\Delta K\in R^{n\times n}$. Then the motion of the modified system is governed by

$$(M_0+\Delta M)\ddot{q}(t)+(K_0+\Delta K)q(t)=f(t) \tag{4}$$

and it satisfies the following eigen-matrix equation:

$$(K_0+\Delta K)Y=(M_0+\Delta M)Y\textstyle\sum.\ \textit{The size of symbol}\ \sum\ \textit{is too big in this edited paper}. \tag{5}$$

where $\sum=\mathrm{diag}(\mu_1,\mu_2,\dots,\mu_n)$ and $Y=(y_1,y_2,\dots,y_n)$ are the complete eigenstructure of the modified system (4).

In [14] a necessary and sufficient condition was proposed for the incremental mass and stiffness matrices that modify some eigenvalues or eigenpairs while keeping other eigenpairs unchanged, which is crucial to address the partial eigenstructure assignment problem by SOF in this paper and thus is shown in the following:

$$\Delta K\left(M_0^{-1}-X_1X_1^T\right)-\Delta M\left(M_0^{-1}K_0M_0^{-1}-X_1\Lambda_1X_1^T\right)=0 \tag{6}$$

Please cite this article as: J. Zhang, et al., Static output feedback for partial eigenstructure assignment of undamped vibration systems, Mech. Syst. Signal Process. (2015), http://dx.doi.org/10.1016/j.ymssp.2015.07.012