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A variable-coefficient harmonic balance method for the prediction of quasi-periodic response in nonlinear systems

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ABSTRACT

Quasi-periodic responses arise from various nonlinear dynamic systems under a single-frequency excitation. A variable-coefficient harmonic balance method is proposed for the prediction of quasi-periodic responses. The key point of this method is that the quasi-periodic response is described as a truncated trigonometric series with time-periodic Fourier coefficients. In other words, quasi-periodic responses are treated in a "cascade" of frequency base. Harmonic terms in the nonlinear system are separated and balanced with respect to each basic frequency. Numerical examples reveal that this method is efficient in predicting such quasi-periodic responses, which contain an unknown frequency component.

1. Introduction

Coupled oscillators occur in many applications ranging from aerodynamics and structural dynamics. The existence of steady, quasi-periodic responses arising from coupled oscillators has been observed in diverse engineering systems [1–4]. In general, a quasi-periodic oscillation that is associated with p different internal frequencies takes place on a p-dimensional invariant torus. Any quasi-periodic response $x(t)$ can be expressed as a function $x(t)=x(\omega t)$. The tuple $\omega=(\omega_1,\dots,\omega_p)$ is called the frequency base. Since the solution $x(t)$ is quasi-periodic, the basic frequencies w_j must be incommensurate (rationally independent), that is, for integers k_j the equation $\langle k,\omega\rangle := \sum_{j=1}^{p} k_j\omega_j = 0$ holds if and only if all $k_j=0$ for $j=1,\dots,p$. For $p=2$ this means that the ratio ω_1/ω_2 is irrational. In this paper, we use the term "quasi-periodic response" for a quasi-periodic response with a two-dimensional frequency base.

Quasi-periodic responses naturally result from quasi-periodically forced nonlinear systems with a two-dimensional frequency base. For example, multiple-rotor systems in aeroengines are subject to multi-frequency excitation due to several coexisting unbalances [1]. For such nonlinear systems, the response possesses the same fundamental frequency base as the external excitation, whose frequency base is determined. The dynamic response can be thus predicted by the numerical algorithm based on a multi-dimensional generalization of harmonic balance [2]. In this method, the quasi-periodic response is represented by a multi-dimensional Fourier series. The problem of predicting a quasi-periodic response is converted into the problem of seeking a solution of an algebraic system of equations. This solution provides the amplitudes associated with the different frequency spectral peaks in the assumed Fourier series. However, a conventional multi-dimensional harmonic

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balancing requires all the basic frequencies as a prerequisite; on the other hand, the concept of hyper-time is used to extend the harmonic balancing to quasi-periodicity; the associated computational cost increases significantly with the number of harmonics.

Apart from the multi-dimensional harmonic balance method, there also exist a number of methods for constructing quasi-periodic solutions [5]. In general, analytical methods such as perturbation methods, multiple scales method and averaging method are limited in small-sized systems possessing certain types of nonlinearities. Poincaré maps play a key role in global analyses; the construction of high-order Poincaré sections depends on the availability of a large amount of data and on the closeness of the ratio of the incommensurate frequencies to a rational number; hence, an adequate knowledge of the nonlinear system (special emphasis on the basic frequencies) is required.

In particular, quasi-periodic responses have also been observed in nonlinear systems undergoing single-frequency excitation. A well-known case is that rotors with bearing clearances and cross-coupling stiffness coefficients may exhibit quasi-periodic response under a mono-harmonic excitation [3,6,7]. A simplified Jeffcott rotor model is usually adopted to gain insights into many important phenomena observed in real applications. The transition from no contact to contact is described by non-smooth nonlinear restoring forces. Cross-coupling stiffness might lead to a new output frequency component, which is responsible for the outcoming of quasi-periodic responses. Research efforts have been focused on frequency domain approaches to obtain steady state responses and understand the transition between periodic and quasi-periodic responses. Provided that the extra-unknown basic frequency can be determined by, e.g., applying FFT algorithm to the time history of the response, the quasi-periodic whirling response can be obtained similarly by means of multi-dimensional harmonic balance method [3] or the so-called quasi-periodic harmonic balance method [8] in conjunction with pseudo arc length continuation. However, methodologies of this kind are initiated by time integration simulations. The prerequisite is to properly approximate the unknown basic frequency with an acceptable computational cost, which remains a challenge in the literature.

Quasi-periodic response has also been widely investigated in the studies of nonlinear energy sinks (NES) in recent years. A SDOF linear oscillator weakly coupled to a local small mass through strong stiffness nonlinearity, which acting as nonlinear energy sink composes the simplest NES system (see Section 3). It is reported that even in the case of single-frequency excitation, quasi-periodic response regimes are demonstrated to typically exist together with periodic response regimes [9]. The new-arising basic frequency due to nonlinear coupling is not determined. In addition to the time marching method, only analytical approach has been proposed for the strongly modulated response description [4].

In this paper, a new numerical approach, called Variable-Coefficient Harmonic Balance Method (VCHBM), will be developed to predict the aforementioned quasi-periodic response precisely. The unknown basic frequency in the quasi-periodic response is not necessarily predetermined, which advantages this new VCHBM over the multi-dimensional harmonic balance method or the so-called quasi-periodic harmonic balance method. This method derives from the classical harmonic balancing and its essential idea is to treat the quasi-periodic response as a truncated trigonometric series with time-periodic coefficients. Harmonic balancing procedure is performed with respect to each basic frequency involved in the quasi-periodic response. These time-periodic coefficients can then be sought by solving an algebraic system of equations resulting from the previous harmonic balancing. As an effective numerical method currently well-developed and optimized even for large scale nonlinear systems, the harmonic balance method is employed for seeking these variable Fourier coefficients. To this end, the conventional harmonic balance method is first presented in Section 2, which serves as the framework of harmonic balancing; then the principle of VCHBM is outlined and practical aspects concerning this numerical algorithm are discussed; numerical examples are given in Section 3 in order to validate this newly developed method; conclusions are drawn in the end.

2. Variable-coefficient harmonic balance method for quasi-periodic response

In this section, a variable-coefficient harmonic balance method is developed to characterize the quasi-response of nonlinear systems. Let us consider a non-autonomous nonlinear dynamic system described by the following second order differential equation:

$$\mathbf{M}\ddot{\mathbf{x}}+\mathbf{D}\dot{\mathbf{x}}+\mathbf{K}\mathbf{x}+\mathbf{f}_{\mathrm{nl}}(t,\mathbf{x},\dot{\mathbf{x}})=\mathbf{f}(t) \tag{1}$$

where $\mathbf{M}$, $\mathbf{D}$ and $\mathbf{K}$ are mass, damping and stiffness matrix of size $n\times n$, respectively; $\mathbf{f}_{\mathrm{nl}}(t,\mathbf{x},\dot{\mathbf{x}})$ represents nonlinear force and $\mathbf{f}(t)$ the external periodic excitation with a single basic frequency ω. Since the Harmonic Balance Method (HBM) is employed as a principal tool in this paper, it will be introduced in detail. Continuation technique is also covered in order to follow the branch of solutions by HBM.

2.1. Harmonic balance method and continuation technique

In majority of cases, periodic response of nonlinear systems is observed. n-dimensional periodic solutions $x(t)$ to Eq. (1) of a unique fundamental frequency ω can be expressed by a truncated Fourier series:

$$\mathbf{x}(t)=\mathbf{X}^0+\sum_{k=1}^{N_h}\mathbf{X}^{ck}\cos(k\omega t)+\mathbf{X}^{sk}\sin(k\omega t) \tag{2}$$