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Nonlinear normal modes of a two degrees-of-freedom piecewise linear system

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ABSTRACT

A study of the Nonlinear Normal Modes (NNMs) of a two degrees of freedom mechanical system with a bilateral elastic stop for one of them is considered. The issue related to the non-smoothness of the impact force is handled through a regularization technique. The Harmonic Balance Method (HBM) with a large number of harmonics, combined with the Asymptotic Numerical Method (ANM), is used to solve the regularized problem. The results are validated from periodic orbits obtained analytically in the time domain by direct integration of the non-regular problem. The first NNM shows an elaborate dynamics with the occurrence of multiple impacts per period, internal resonance and instabilities. On the other hand, the second NNM presents a more simple, almost linear, dynamics. The two NNMs converge asymptotically (for an infinite energy) toward two other Linear Normal Modes, corresponding to the system with a gap equal to zero.

1. Introduction and industrial issue

Many engineering systems involve components with clearance and intermittent contact. This type of nonlinearities is relevant for example in nuclear power plants, specifically in steam generator. In vibration analysis this type of nonlinearities can be modeled considering piecewise linear elastic stops [29,8] or nonlinear elastic stops [7] or rigid impacts [16]. Such nonsmooth systems have been subject of numerous investigations specially to analyze forced responses. The following references [31,35,1,9] give a small selection of the developed procedures.

Recent works have shown that the Nonlinear Normal Modes (NNMs) constitute an efficient vibration analysis framework for nonlinear mechanical systems from theoretical [34,15] as well as experimental [24] point of view. The NNMs can be viewed as an extension of the concept of the normal modes in the theory of the linear systems to nonlinear ones. One of the most attractive definitions is due to Shaw and Pierre [30] in terms of a two-dimensional invariant manifold in phase space. This definition has the advantage that it is valid for conservative and non-conservative systems. However in case of conservative systems, a more numerically tractable definition can be used. This definition is an extension of the definition introduced by Rosenberg [27] and considers a NNM as a family of free motion parametrized by energy level. Hence, the

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NNMs can be computed using numerical continuation techniques of periodic solutions in conservative system. An approach combining a shooting method to approximate the periodic orbit in time domain and the pseudo-arclength continuation method is proposed in [25] to compute the NNMs. Another methodology combining the Harmonic Balance Method (HBM) to approximate the periodic orbit and the Asymptotic Numerical Method (ANM) [4] as a continuation method is discussed in [6,14] to compute the periodic solutions of dynamical systems and can be advantageously used to compute the NNMs. The first step of this methodology is to recast the dynamical system in quadratic polynomial form before applying the HBM resulting in optimal form to apply the ANM method. In [6], polynomial nonlinearities are considered whereas in [14] the procedure is extended to non-polynomial nonlinearities. Other approaches exist like combining the alternating frequency/time-domain harmonic balance method (AFT-HBM) with the pseudo-arclength continuation method to compute mostly forced response [12,9]. Note that in the last two references, the selection procedures of the number of harmonics in the HBM procedure are proposed and investigated.

The concept of NNMs is not limited to smooth systems. Nonsmooth systems have received great attention in regard to NNMs. Conservative piecewise linear vibratory systems were considered in [3,13] where NNMs were obtained using the invariant manifolds form. In [21] the concept of NNMs formulated as a functional relation between the two coordinates of the system was used to analyze a two Degrees Of Freedom (DOF) system with vibro-impact allowing the computation of various branches of bifurcating periodic solutions with different impacting characteristics. Rigid elastic stops were considered in [33] where the NNMs of a single DOF linear system with a vibro-impact attachment were obtained by employing the method of nonsmooth transformation introduced in [26] to approximate the periodic orbit in time domain. In [23], the family of periodic solutions were found in analytical form for a conservative two DOF oscillator with elastic and rigid stop. In [17], a dissipative system is considered and a Fourier series including decreasing exponential terms is used to approximate the NNMs combined with a HBM formulation.

Though many researchers have examined the problem of computing the nonlinear normal modes for nonsmooth systems, few tools to analyze the complete behavior of the NNMs including bifurcation diagram analysis, internal resonances characterization and stability properties are available. In this context, the objective of this paper is to demonstrate that a method combining the HBM method (to approximate the periodic responses) and the ANM (to carry out the continuation of branches of periodic orbits) to analyze the NNMs of a nonsmooth system can be efficient. As suggested in [6], the efficiency of the HBM and ANM will be ensured introducing a regularization of the nonsmooth terms using a family of implicit polynomial. A two degrees of freedom oscillator with a bilateral elastic stop have been used to carry out the study. It is an analogy with a simplified model of the out-of-plane bending of U-tube going through supporting plates as shown in Fig. 1. Similar two degrees-of-freedom systems have been considered in [3,13,23].

The paper is organized as follows. In the next section, the nonsmooth model under consideration is described and periodic orbits with two impacts per period are investigated. Section 3 is dedicated to the description of the proposed procedure to compute the NNM branches. A regularized model is first introduced and the HBM and ANM methods are next described. Finally, in Section 4, the results for the NNMs are discussed in detail, analyzing the influence of the regularization parameters.

2. Nonsmooth model under consideration

2.1. A two degrees of freedom oscillator with an elastic stop

The system under consideration is shown Fig. 1 (right side). It consists of two masses m_1 and m_2 connected by two linear springs of stiffness k_1 and k_2. The motion of the mass m_1 is limited by a bilinear elastic stop with a linear spring of stiffness K

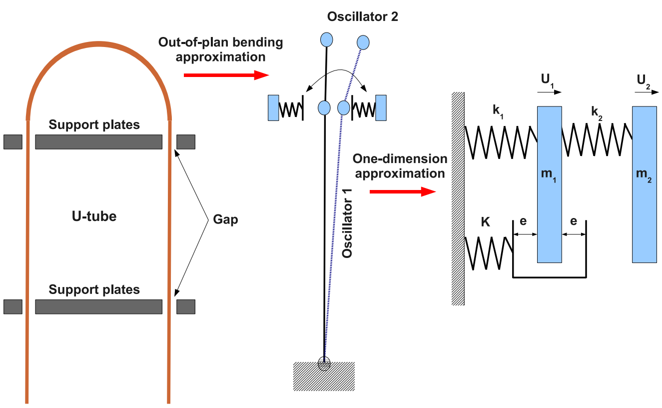

Fig. 1. Analogy of a simplified model of a U-tube with a two degrees of freedom oscillator.