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The envelope-based cyclic periodogram

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ABSTRACT

Cyclostationary analysis has proven effective in identifying signal components for diagnostic purposes. A key descriptor in this framework is the cyclic power spectrum, traditionally estimated by the averaged cyclic periodogram and the smoothed cyclic periodogram. A lengthy debate about the best estimator finally found a solution in a cornerstone work by Antoni, who proposed a unified form for the two families, thus allowing a detailed statistical study of their properties. Since then, the focus of cyclostationary research has shifted towards algorithms, in terms of computational efficiency and simplicity of implementation.

Traditional algorithms have proven computationally inefficient and the sophisticated "cyclostationary" definition of these estimators slowed their spread in the industry. The only attempt to increase the computational efficiency of cyclostationary estimators is represented by the cyclic modulation spectrum. This indicator exploits the relationship between cyclostationarity and envelope analysis. The link with envelope analysis allows a leap in computational efficiency and provides a "way in" for the understanding by industrial engineers. However, the new estimator lies outside the unified form described above and an unbiased version of the indicator has not been proposed.

This paper will therefore extend the analysis of envelope-based estimators of the cyclic spectrum, proposing a new approach to include them in the unified form of cyclostationary estimators. This will enable the definition of a new envelope-based algorithm and the detailed analysis of the properties of the cyclic modulation spectrum. The computational efficiency of envelope-based algorithms will be also discussed quantitatively for the first time in comparison with the averaged cyclic periodogram. Finally, the algorithms will be validated with numerical and experimental examples.

1. Introduction

1.1. Cyclostationarity and cyclic power spectrum

The framework of cyclostationarity has been demonstrated very effective in describing vibration signals recorded on rotating machines. In fact, most of these signals are stochastic (*i.e.* it is not possible to predict the exact value of future samples) but are generated from instantaneous distributions (*i.e.* probability densities for each time instant) whose statistical moments show specific periodicities, strictly linked to the operational speed of the machine. Signals with periodic second order statistics (*i.e.* covariance function and variance) are of particular interest for the diagnostic of rotating and alternating machine components.

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Symbol list

General symbolism

$\mathscr{F}^{(Q)}_{u,p_1,\ldots,p_n}[k](w)$ Quantity $\mathscr{F}$, calculated for signal u using method Q. The quantity is calculated using parameters $p_1,\ldots p_n$ and is a function of the variable u, discretised with resolution Δ_u ($u=k\Delta_u, k\in\mathbb{Z}$), and w, which is continuous

Variables and domains

t time, continuous
Δ_t sampling interval of a discretely sampled time domain signal
$n\Delta_t$ discretised time $n\in\mathbb{Z}$
$\tau\Delta_t$ discretised time domain lag (for correlation/convolution), $\tau\in\mathbb{Z}$
f continuous frequency domain (spectral)
Δ_f frequency resolution of a discrete frequency domain (spectral)
$b\Delta_f$ discretised frequency (spectral) $b\in\mathbb{Z}$
α continuous frequency domain (cyclic)
Δ_α frequency resolution of a discrete frequency domain (cyclic)
$a\Delta_\alpha$ discretised frequency (cyclic) $a\in\mathbb{Z}$
α_0 fundamental cyclic frequency, corresponding to the cyclic period of second order cyclostationary signal
$a\alpha_0$ cyclic frequency corresponding to the ath component of a Fourier series decomposition of the cyclic correlation function of a second order cyclostationary signal with fundamental cyclic period $1/a_0$
f_{cut} cutting frequency (low-pass filter)

Signals and signal processing basics

$x[n]$ discrete sampled signal $x[n]=x(n\Delta_t)$ (sampling freq. $1/\Delta_t$)
$w[n]$ windowing function (discrete time domain)
$g[\tau]$ smoothing discrete time function (function of the time lag $\tau\Delta_t$)
$X_L(f)$ result of the DTFT of an L-sample long signal $x[n]$, *i.e.* $n=1,\ldots,L$. Continuous frequency axis f (no FFT)
$X^{(w)}_L[h]$ result of the FFT of an L-sample long signal $x[n]$, *i.e.* $n=1,\ldots,L$. The superscript (w) is present when a windowing function $w[n]$ is applied. Frequency resolution is $h/(\Delta_t L)$
$X^{(w)}_{L,N,R}[k](f)$ spectrogram of the L-sample long signal $x[n]$, obtained by DTFT operations over N-sample long, w-windowed subrecords of with overlap among of $(N-R)$ samples. Continuous frequency axis f (no FFT), discrete time domain with resolution $R\Delta_t$
$X^{(w)}_{L,N,R}[k,b]$ spectrogram of the L-sample long signal $x[n]$, obtained by FFT operations over N-sample long w-windowed subrecords with overlap $(N-R)$ samples. Time domain resolution is $R\Delta_t$ (index k), frequency resolution Δ_f (index b)
$x_L[an]$ $(a\Delta_\alpha)$-frequency shifted version of an L-sample long signal $x[n]$. The shift is repeated with a frequency resolution of Δ_α
$X^{(w)}_{L,N,R}[k,a,b]$ spectrogram of $x_L[a,n]$, obtained by FFT operations over N-sample long w-windowed subrecords with overlap $(N-R)$ samples. Time resolution is $R\Delta_t$ (index k), cyclic freq. resolution Δ_α (index a), spectral freq. resolution Δ_f (index b)

Statistical quantities

$\mathscr{R}_w[\tau]$ autocorrelation function of the windowing function $w[n]$
$\mathscr{R}_{2x}[n,\tau]$ instantaneous autocorrelation function the discrete sampled signal $x[n]$ at time $n\Delta_t$ with lag $\tau\Delta_t$
$\mathscr{R}_{2x}[a,\tau]$ Fourier coefficient of $\mathscr{R}_{2x}[n,\tau]$, assuming α_0 as the fundamental frequency of the second order cyclostationary signal $x[n]$
$\mathcal{S}_{2x}[a](f)$ cyclic power spectrum of the signal $x[n]$. Cyclic frequency axis has resolution α_0 (index a), spectral frequency axis f continuous

Estimators

$\hat{\mathscr{R}}_{2x,L}[\tau](\alpha)$ estimator for the Fourier coefficient of the instantaneous autocorrelation function (lag $\tau\Delta_t$) at the cyclic frequency α
$\hat{\mathcal{S}}^{()}_{2x,L}(\alpha,f)$ general estimator of the cyclic power spectrum for the signal $x[n]$, using a L-samples long record of the signal
$\hat{\mathcal{S}}^{(P)}_{2x,L}(\alpha,f)$ cyclic periodogram
$\hat{\mathcal{S}}^{(g)}_{2x,L}(\alpha,f)$ smoothed cyclic periodogram with smoothing function $g[\tau\Delta_t]$
$\hat{\mathcal{S}}^{(w)}_{2x,L,N}(\alpha,f)$ averaged cyclic periodogram obtained with windowing function w and subrecords of length N (continuous version, no FFT)
$\hat{\mathcal{S}}^{(w)}_{2x,L,N,R}[a,b]$ averaged cyclic periodogram obtained with windowing function w and subrecords of length N (discrete version allows FFT) with overlap $(N-R)$. Cyclic freq. resolution Δ_α (index a), spectral freq. resolution Δ_f (index b)
$\mathrm{CMS}^{(w)}_{2x,L,N,R}[a,b]$ cyclic modulation spectrum obtained with windowing function w and subrecords of length N (discrete version allows FFT) with overlap $(N-R)$. Cyclic freq. resolution Δ_α (index a), spectral freq. resolution Δ_f (index b)
$\hat{\mathcal{S}}^{(h)}_{2x,L,N,R}[a,b]$ estimator of the cyclic power spectrum based on the cyclic modulation spectrum. Cyclic freq. resolution Δ_α (index a), spectral freq. resolution Δ_f (index b)
$\hat{\mathcal{S}}^{(g)}_{2x,L,N,R}[a,b]$ estimator of the cyclic power spectrum based on the cyclic modulation spectrum with Gaussian window. Cyclic freq. resolution Δ_α (index a), spectral freq. resolution Δ_f (index b)