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Brief paper

Receding horizon control based consensus scheme in general linear multi-agent systems ☆

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ABSTRACT

This paper investigates the consensus problem of general linear multi-agent systems under the framework of optimization. A novel distributed receding horizon control (RHC) strategy for consensus is proposed. We show that the consensus protocol generated by the unconstrained distributed RHC can be expressed in an explicit form. Based on the resulting consensus protocol the necessary and sufficient conditions for ensuring consensus are developed. Furthermore, we specify more detailed consensus conditions for multi-agent systems with general and one-dimensional linear dynamics depending on Riccati difference equations (RDEs), respectively. Finally, a case study verifies the proposed scheme and the corresponding theoretical results.

1. Introduction

In the last two decades, the cooperative control of networked multi-agent systems has received a lot of attention due to its wide applications. In particular, the consensus problem is of significant importance, and has inspired much progress, e.g., Moreau (2005), Olfati-Saber and Murray (2004) and Ren and Beard (2005). In this paper, we are interested in solving the consensus problem of multi-agent systems from the distributed optimal control perspective. The multi-agent system under study is of fixed directed network topology and general linear time invariant (LTI) dynamics associated with each agent. The objective of this paper is to design a locally optimal consensus strategy for each agent, and further to investigate under what conditions the closed loop system can achieve consensus by the designed strategy.

The optimality of control protocols brings many desired properties such as phase and gain margin, leading to robustness of the closed loop systems. The core difficulty of the cooperative optimal control for multi-agent systems lies in the fact that the centralized optimization problem generally cannot be distributed among agents, with few exceptions (Cao & Ren, 2010; Hengster-Movric & Lewis, 2014). As a result, the best way of circumventing the difficulty is to adopt the locally optimal control strategy and utilize regional information to address the system-level interaction and coupling, approximately achieving some global or cooperative behaviors.

In the literature, one approach to the optimal cooperative control is the linear quadratic regulation (LQR) scheme. For example, the distributed LQR problem of multi-agent systems with LTI dynamics is studied in Borrelli and Keviczky (2008), showing that the overall stability can be guaranteed by appropriately designing the local LQR and using information exchanged over networks. The consensus problem with optimal Laplacian matrix for multi-agent systems of first-order dynamics is investigated in Cao and Ren (2010), where it is shown that the globally optimal Laplacian matrix can only be obtained by properly choosing the global cost function coupled with the network topology. Recently, the LQR-based consensus problem of multi-agent systems with LTI dynamics and fixed directed topology is addressed in Hengster-Movric and Lewis (2014), indicating that the globally optimal consensus performance can be achieved by using locally optimal consensus protocol if and only if the overall performance index is selected in a special form depending on the network structure.

Another way of achieving the (sub-)optimal cooperative control of multi-agent systems is the distributed receding horizon control (RHC) strategy, also known as distributed model predictive

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control. Based on this approach, there have been many results developed for cooperative stabilization, formation control, and its applications. For example, the distributed RHC-based scheme for cooperative stabilization is proposed in Dunbar and Murray (2006) and Müller, Reble, and Allgöwer (2012), and the formation stabilization is addressed in Keviczky, Borrelli, and Balas (2006) and its application is reported in Keviczky, Borrelli, Fregene, Godbole, and Balas (2008). Furthermore, the robust distributed RHC problems that can be used for cooperative stabilization are studied in Richards and How (2007) for linear systems with coupled constraints and in Li and Shi (2014a) for nonlinear systems. To further attack the unreliability of the communication networks, the cooperative stabilization problem of multi-agent nonlinear systems with communication delays are investigated in Franco, Magni, Parisini, Polycarpou, and Raimondo (2008) and Li and Shi (2013, 2014b). Note that all of these results use cost functions as Lyapunov functions to prove stability.

Even though it is very desirable to achieve optimal consensus by distributed RHC scheme, there have been few results for the consensus problem of multi-agent systems due to the difficulty that the cost function may not be directly used as Lyapunov function. In Ferrari-Trecate, Galbusera, Marciandi, and Scattolini (2009), Ferrari-Trecate et al. study the consensus problem of multi-agent systems of first-order and second-order dynamics, and the sufficient conditions for achieving consensus are developed by exploiting the geometric properties of the optimal path. Zhan et al. investigate the consensus problem of first-order sampled-data multi-agent systems in Zhan and Li (2013), where the state and control input information needs to be exchanged. These two results are effective to deal with special types of linear systems. In Johansson, Speranzon, Johansson, and Johansson (2008) Johansson et al. propose to use the negotiation to reach the optimal consensus value by implementing the primal decomposition and incremental sub-gradient algorithm, but the effect of the network topology is not explicitly considered.

It can be seen that the receding horizon control-based consensus problem for multi-agent systems with general LTI dynamics has not been solved, and the relationship between consensus and the interplay between the network topology and the RHC design is still unclear, which motivates this study. The main contribution of this paper is two-fold.

- A novel distributed RHC strategy is proposed for designing the consensus protocol. In this strategy, each agent at each time instant only needs to obtain its neighbors' state once per step via communication network, which is more efficient than the work in Dunbar and Murray (2006) and Li and Shi (2014a) (where the state and its predicted trajectory need to be transmitted) and in Müller et al. (2012) and Zhan and Li (2013) (where the neighbors' information needs to be exchanged for many times at each time instant). In addition, we show that the consensus protocol generated by the RHC is a feedback of the linear combination of each agent's state and its neighbors' states, and the feedback gains depend on a set of difference matrix equations. This result partially extends the results in Ferrari-Trecate et al. (2009) and Zhan and Li (2013) to multi-agent systems with LTI dynamics.
- Given the proposed distributed RHC strategy, a necessary and sufficient condition for ensuring consensus is developed. We show that, under some mild assumptions, the consensus can be reached if and only if a simultaneous stabilization problem can be solved. Furthermore, specific sufficient consensus conditions depending on the Riccati difference equation (RDE) for multi-agent systems with LTI dynamics and one-dimensional linear dynamics are also developed, respectively.

The remainder of this paper is organized as follows. Section 2 introduces some well-known results of graph theory and formulates the problem to be studied. Section 3 presents the novel distributed RHC scheme, and develops a detailed consensus protocol. The necessary and sufficient conditions for ensuring consensus are proposed in Section 4, and more specific sufficient consensus conditions for multi-agent systems with LTI dynamics and one-dimensional linear dynamics are also reported in this section. The case study and comparisons are demonstrated in Section 5. Finally, the conclusion remarks are summarized in Section 6.

Notation. The symbol $\mathbb{R}$ represents the real numbers. For a real matrix A, its transposition and inverse (if the inverse exists) are denoted as "A^{T}" and "A^{-1}", respectively. If A is a complex matrix, then the transposition is denoted by A^{H}. Given a real (or complex) number λ, the absolute value (modulus) is defined as $|\lambda|$. Given a matrix (or a column vector) X and another matrix P with appropriate dimension, the 2-induced norm (or the Euclidean norm) of X is denoted by $\|X\|$ and the P-weighted norm of X is denoted by $\|X\|_P \triangleq \sqrt{X^{\mathrm{T}}PX}$. Given a matrix Q, $Q > 0$ ($Q \geqslant 0$) stands for the matrix Q being positive definite (semi-positive definite). Define the column operation $\mathrm{col}\{x_1, \dots, x_p\}$ as $[x_1^{\mathrm{T}}, \dots, x_p^{\mathrm{T}}]^{\mathrm{T}}$, where $x_1, \dots, x_p$ are column vectors. I_p stands for the identity matrix of dimension p, and $\mathbf{1}_p$ and $\mathbf{0}_p$ represent the p-dimensional column vector $[1, \dots, 1]^{\mathrm{T}}$ and $[0, \dots, 0]^{\mathrm{T}}$, respectively. The symbol $\otimes$ stands for the Kronecker product.

2. Problem formulation

Consider a multi-agent system of M linear agents. For each agent i, the dynamics is described as

$$x_i(k+1) = Ax_i(k) + Bu_i(k), \tag{1}$$

where $x_i(k) \in \mathbb{R}^p$ is the system state, and $u_i(k) \in \mathbb{R}^m$ is the control input.

There exists a communication network among the M-agent system, and the network topology is described as a directional graph (digraph) $\mathcal{G} \triangleq \{\mathcal{V}, \mathcal{E}, \mathcal{A}\}$. Here, $\mathcal{V} = \{i, i = 1, \dots, M\}$ is the collection of the nodes of the digraph representing each agent i, $\mathcal{E} \subset \mathcal{V} \times \mathcal{V}$ denotes the edges of paired agents, and $\mathcal{A} = [a_{ij}] \in \mathbb{R}^{M\times M}$ is the adjacency matrix with $a_{ij} \geqslant 0$. If there is a connection from agent j to i, then $a_{ij} = 1$; otherwise, $a_{ij} = 0$. We assume there is no self-edge in the digraph $\mathcal{G}$, i.e., $a_{ii} = 0$. For each agent i, its neighbors are denoted by the agents from which it can obtain information, and the index set for agent i's neighbors is denoted as $\mathcal{N}_i \triangleq \{j|(i, j) \subset \mathcal{E}\}$, where the pair (i, j) represents that agent i is connected to j and can directly obtain information from j. The number of agents in $\mathcal{N}_i$ is denoted as $|\mathcal{N}_i|$. The graph $\mathcal{G}$ contains a spanning tree if and only if there exists a root node that can send information to all the other agents through directed paths. The degree matrix is denoted as $\mathcal{D} = \mathrm{diag}(\sum_{j=1}^{M} a_{1j}, \dots, \sum_{j=1}^{M} a_{Mj})$, and the Laplacian matrix of $\mathcal{G}$ is denoted as $\mathcal{L} = \mathcal{D} - \mathcal{A}$. Arrange the eigenvalues of $\mathcal{L}$ as $|\lambda_1| \leqslant |\lambda_2| \leqslant \cdots \leqslant |\lambda_M|$. Assume that the digraph $\mathcal{G}$ is fixed. Firstly, we recall a standing result from the graph theory (Olfati-Saber & Murray, 2004; Ren & Beard, 2005; You & Xie, 2011).

Lemma 1. *The digraph $\mathcal{G}$ contains a spanning tree if and only if zero is a simple eigenvalue of the Laplacian matrix $\mathcal{L}$, i.e., $\lambda_1 = 0 < |\lambda_2| \leqslant \cdots \leqslant |\lambda_M|$, and the corresponding right eigenvector of λ_1 is $\mathbf{1}$.*

For the linear system in (1), a standing assumption is made: The pair $[A, B]$ is controllable. We assume that at each time instant k, over the given communication network, agent i can get state information $x_j(k), j \in \mathcal{N}_i$ from its neighbors. The communication network is reliable and the information can be transmitted instantaneously without time consumption.