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Brief paper

Bounded synchronization of a heterogeneous complex switched network ☆

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ABSTRACT

This paper investigates synchronization issues of a heterogeneous complex network with a general switching topology in the sense of boundedness, when no complete synchronization manifold exists. Several sufficient conditions are established with the Lyapunov method and the differential analysis of convergence to determine the existence and estimate the convergence domain for the local and global bounded synchronization of a heterogeneous complex network. By using the consensus convergence of a switched linear system associated with the switching topology, explicit bounds of the maximum deviation between nodes are obtained in the form of a scalar inequality involving the property of the consensus convergence, the homogeneous and heterogeneous dynamics of individual nodes for the local and global cases. These analytical results are simple yet generic, which can be used to explore synchronization issues of various complex networks. Finally, a numerical simulation illustrates their effectiveness.

1. Introduction

Synchronization of populations of locally interacting units is an active field of research with applications in science and engineering, see Arenas, Díaz-Guilera, Kurths, Moreno, and Zhou (2008), Osipov, Kurths, and Zhou (2007), Pikovsky, Rosenblum, and Kurths (2001) and references therein. From the viewpoint of complex network, synchronization of complex systems is usually determined by the dynamics of individual nodes and the coupling configuration between nodes. This result has been established by mainly assuming that all the node dynamics are identical, see Belykh, Belykh, and Hasler (2006), Stilwell, Bollt, and Roberson (2006), Wu (2005), Yu et al. (2009) and Zhao, Hill, and Liu (2009). However, significant differences commonly exist within the relevant individual nodes. Motivated by this, we investigate synchronization issues of complex heterogeneous networks in this paper.

The behavior of complex dynamical networks with nonidentical nodes is much more complicated than that of the identical-node case since synchronization manifold, guaranteed by the diffusive condition in identical-node networks, disappears due to the heterogeneity of individual nodes. The ultimate synchronous trajectory, in general, has to be confined to some particular solution. Also, decompositions into a few of lower dimensional subsystems are no longer possible even for local synchronization of heterogeneous networks. Thus, it is quite difficult to explore synchronization of complex heterogeneous networks, and very few results have been reported to date. A simple case for all nonidentical nodes shared with a common equilibrium has been studied in Xiang and Chen (2007) and Zhao, Hill, and Liu (2011). Besides, synchronization of coupled nonidentical chaotic systems have also been discussed in Duan and Chen (2009), Femat, Kocarev, van Gerven, and Monsivais-Pérez (2005) and Li, Chen, and Aihara (2006).

There is no doubt that a complex network of coupled nonidentical systems may still exhibit some kind of synchronous behaviors that need to be understood. Bounded synchronization is a typical weaker form of synchronization when complete synchronization is impossible. Examples include clock synchronization in mobile robots, or task coordination of swarming animals, or the appearance of synchronized bulk oscillations in suspension of yeast

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cells, etc. Several investigations have discussed the bounded synchronization issues, e.g., in stochastic complex networks (Shen, Wang, & Liu, 2011) and consensus control of multi-agent systems (Zhong, Liu, & Thomas, 2012). A very recent result in Zhao, Hill, and Liu (2012) has addressed synchronization of a general dynamical network with nonidentical nodes and symmetric coupling matrix. Wang, Qian, and Wang (2015) introduced a generalized connection graph stability method to avoid the calculating eigenvalues of an asymmetry coupling matrix. It is noted that the above mentioned topology has dealt with a "slow-varying" structure, i.e., a continuous function is simply taken as the varying connection for a time-varying network. Such a description has limitations in handling the varying topologies that change very quickly such as switches due to connection failures and new creations. In general, the relevant time-varying results exhibit strong conservativeness, particularly in analyzing an unconnected topology. A few recent results (Frasca, Buscarino, Rizzo, Fortuna, & Boccaletti, 2008; Wang, Shi, & Sun, 2008) have discussed switching synchronization issues of a mobile agent network under the fast-switching constraint. More results on switching networks have focused on the consensus of multi-agent systems from the viewpoint of control science, see Olfati-Saber and Murray (2004) and Ren and Beard (2005) for example. This paper investigates bounded synchronization of a heterogeneous complex switched network. With the Lyapunov function approach as well as differential analysis of convergence, we derive several bounded synchronization conditions for the heterogeneous network. In particular, the problem of bounded synchronization under the general switching topology is solved by partially using the techniques of the consensus problem because of the similarity of synchronization in complex coupled networks and consensus in multi-agent systems.

The rest of this paper is organized as follows. Section 2 presents a heterogeneous complex switched network and some mathematical preliminaries. In Section 3, we derive several sufficient conditions to guarantee the local and global bounded synchronization of the considered network, respectively. A numerical example is given to elucidate the effectiveness of the presented results in Section 4. Section 5 concludes the investigation.

2. A heterogeneous network model

Consider a nonlinear system of N linearly and diffusively coupled nonidentical nodes which are represented by

$$\dot{\mathbf{x}}_i(t) = \mathbf{f}_i(t, \mathbf{x}_i) - c\sum_{j=1}^{N} L_{\sigma ij}\Gamma\mathbf{x}_j(t), \quad i = 1, 2, \ldots, N, \tag{1}$$

where $\mathbf{x}_i = [x_{i1}, x_{i2}, \ldots, x_{in}]^{\mathrm{T}} \in \mathbb{R}^n$ is the state vector of node i, $\mathbf{f}_i : [0, \infty) \times D \to \mathbb{R}^n$ is continuously differentiable with $D \subseteq \mathbb{R}^n$, governing the dynamics of each node, $c > 0$ is the overall coupling strength, $\Gamma = \mathrm{diag}(\gamma_1, \ldots, \gamma_n) \in \mathbb{R}^{n\times n}$ is the inner-coupling matrix, the switching signal $\sigma(t) : [0, \infty) \to \mathcal{P} = \{1, 2, \ldots, p\}$ with $p < \infty$ is a piecewise constant function with successive times to describe the topology switches between subintervals. In network (1), the communication topology is represented by digraph $\mathcal{G}^{\sigma}$ and described in a matrix form by the Laplacian $L_{\sigma(t)} = (L_{\sigma ij}) \in \mathbb{R}^{N\times N}$. The Laplacian of digraph $\mathcal{G}^{\sigma}$ is defined as follows: If there is a directed connection from node j to node i ($i \neq j$) at time t, then $L_{\sigma ij} < 0$; otherwise $L_{\sigma ij} = 0$, and its diagonal entries satisfy $L_{\sigma ii} = -\sum_{j=1, j\neq i}^{N} L_{\sigma ij}$, $\forall i$, and $\sigma(t)$.

For simplicity, let $\mathcal{G} = \{\mathcal{G}^i | i \in \mathcal{P}\}$ denote the set of all possible communication graphs of network (1) in the process of switching, each of which represents a digraph with the Laplacian L_i for $i \in \mathcal{P}$. Also, consider an infinite sequence of nonempty, bounded and contiguous time intervals $[t_k, t_{k+1})$, $k = 0, 1, \ldots,$ with $t_0 = 0$ and $t_{k+1} - t_k \le T_{\max}$ for some positive constant $T_{\max}$. In each interval $[t_k, t_{k+1})$, there is a sequence of non-overlapping subintervals $[t_{k_0}, t_{k_1}), [t_{k_1}, t_{k_2}), \ldots, [t_{k_{m_k-1}}, t_{k_{m_k}})$ with $t_{k_0} = t_k$, $t_{k_{m_k}} = t_{k+1}$ satisfying $t_{k_{j+1}} - t_{k_j} \ge T_{\min}$, $0 \le j \le m-1$, for some integer $m \ge 1$ and a given positive constant $T_{\min}$. In particular, the digraph $\mathcal{G}^{\sigma}$ with the Laplacian L_{σ} switches at t_{k_l} and it does not change during each subinterval $[t_{k_l}, t_{k_{l+1}})$. Throughout this paper, notations for graphs and their corresponding Laplacian matrices are not differentiated unless stated otherwise.

Assumption 1 (*A1*)**.** The dynamics of each isolated node can be expressed in the form of $\mathbf{f}_i(t, \mathbf{x}_i) = \mathbf{f}(t, \mathbf{x}_i, \mathbf{X}) + \mathbf{g}_i(t, \mathbf{X})$ and $\|\mathbf{g}_i\| \le \delta$ holds uniformly for all nodes with constant δ as the heterogeneity parameter, where $\mathbf{f} : [0, \infty) \times D \times \cdots \times D \to \mathbb{R}^n$, $\mathbf{g}_i : [0, \infty) \times D \times \cdots \times D \to \mathbb{R}^n$, $\mathbf{X} = [\mathbf{x}_1^{\mathrm{T}}, \mathbf{x}_2^{\mathrm{T}}, \ldots, \mathbf{x}_N^{\mathrm{T}}]^{\mathrm{T}} \in \mathbb{R}^{nN}$, and $\|\cdot\|$ denotes the Euclidean norm.

Note that the heterogeneous dynamics $\mathbf{g}_i(t, \mathbf{X})$ represents the differences arising from the individual nodal dynamics. A common choice of $\mathbf{f}$ is $\mathbf{f}(t, \mathbf{x}_i, \mathbf{X}) = \sum_{i=1}^{N} \xi_i \mathbf{f}_i(t, \mathbf{x}_i)$, where $\xi_i \ge 0$ for all i and $\sum_{i=1}^{N} \xi_i = 1$. Sometimes, one can simply select $\mathbf{f}(t, \mathbf{x}_i, \mathbf{X}) = \mathbf{f}(t, \mathbf{x}_i)$ according to the nodal dynamics for some i. Then, $\mathbf{g}_i(t, \mathbf{X}) = \mathbf{f}_i(t, \mathbf{x}_i) - \mathbf{f}(t, \mathbf{x}_i)$, and the estimation δ associates with the states of nodes. It is noted that δ can be analytically calculated for many coupled limit-cycle or chaotic systems as if the bound of nodal states is known as *a priori*. Besides, the function $\mathbf{g}_i(t, \mathbf{X})$ can also take the noise or external disturbances into account.

Now, let $\mathbf{F}(t, \mathbf{X}) = [\mathbf{f}^{\mathrm{T}}(t, \mathbf{x}_1, \mathbf{X}), \ldots, \mathbf{f}^{\mathrm{T}}(t, \mathbf{x}_N, \mathbf{X})]^{\mathrm{T}} \in \mathbb{R}^{nN}$, $\mathbf{G}(t) = [\mathbf{g}_1^{\mathrm{T}}(t, \mathbf{X}), \ldots, \mathbf{g}_N^{\mathrm{T}}(t, \mathbf{X})]^{\mathrm{T}} \in \mathbb{R}^{nN}$. Then, network (1) can be rewritten in a block form as

$$\dot{\mathbf{X}}(t) = \mathbf{F}(t, \mathbf{X}) - c(L_{\sigma} \otimes \Gamma)\mathbf{X}(t) + \mathbf{G}(t), \tag{2}$$

where $\otimes$ is the Kronecker product. From A1, $\mathbf{G}(t) \equiv 0$ means that the heterogeneity among the node dynamics disappears. Then, network (2) reduces to a complex network of coupled identical nodes

$$\dot{\mathbf{X}}(t) = \mathbf{F}(t, \mathbf{X}) - c(L_{\sigma} \otimes \Gamma)\mathbf{X}(t). \tag{3}$$

For network (3), there always exists an invariant synchronization manifold $\mathbf{S} = \{(\mathbf{x}_1^{\mathrm{T}}, \ldots, \mathbf{x}_N^{\mathrm{T}})^{\mathrm{T}} \in \mathbb{R}^{nN} : \mathbf{x}_i = \mathbf{x}_j, \ \forall i, j\}$. We denote an orthonormal basis of $\mathbf{S}$ by $\tilde{\mathbf{v}} \triangleq (\mathbf{v}^{\mathrm{T}} \otimes I_n) \in \mathbb{R}^{n\times nN}$ and a basis of orthocomplement space of $\mathbf{S}$ (denoted by $\mathbf{S}^{+}$) by $\tilde{\mathbf{v}}_{+} \triangleq (\mathbf{v}_{+}^{\mathrm{T}} \otimes I_n) \in \mathbb{R}^{n(N-1)\times nN}$, where $\mathbf{v} = \frac{1}{\sqrt{N}}[1, 1, \ldots, 1]^{\mathrm{T}} \in \mathbb{R}^N$, and $\mathbf{v}_{+} \in \mathbb{R}^{N\times(N-1)}$. Then, it is easy to verify that $\mathbf{X} \in \mathbf{S} \Longleftrightarrow \tilde{\mathbf{v}}_{+}\mathbf{X} = 0$. Left-multiplying Eq. (2) by $\tilde{\mathbf{v}}_{+}$ yields

$$\dot{\mathbf{Y}}(t) = \tilde{\mathbf{v}}_{+}\mathbf{F}(t, \tilde{\mathbf{v}}_{+}^{\mathrm{T}}\mathbf{Y}(t) + \overline{\mathbf{X}}(t)) - c(\tilde{L}_{\sigma} \otimes \Gamma)\mathbf{Y}(t) + \tilde{\mathbf{v}}_{+}\mathbf{G}(t), \tag{4}$$

where I_n is an $n \times n$ identity matrix, $\mathbf{Y} = \tilde{\mathbf{v}}_{+}\mathbf{X} \in \mathbb{R}^{n(N-1)}$, $\overline{\mathbf{X}} = [\overline{\mathbf{x}}^{\mathrm{T}}, \overline{\mathbf{x}}^{\mathrm{T}}, \ldots, \overline{\mathbf{x}}^{\mathrm{T}}]^{\mathrm{T}} \in \mathbb{R}^{nN}$, $\overline{\mathbf{x}} = \frac{1}{\sqrt{N}}\tilde{\mathbf{v}}\mathbf{X}$ is the average state trajectory, and $\tilde{L}_{\sigma} = \mathbf{v}_{+}^{\mathrm{T}} L_{\sigma}\mathbf{v}_{+}$. Obviously, $\mathbf{Y}(t) = 0$ is an equilibrium point of system

$$\dot{\mathbf{Y}}(t) = \tilde{\mathbf{v}}_{+}\mathbf{F}(t, \tilde{\mathbf{v}}\mathbf{Y}(t) + \overline{\mathbf{X}}(t)) - c(\tilde{L}_{\sigma} \otimes \Gamma)\mathbf{Y}(t). \tag{5}$$

The exponential stability of system (5) is equivalent to the exponential synchronization of network (3) (Wang & Wang, 2013). Correspondingly, the bounded synchronization of network (2) can be assessed by the ultimate boundedness of the solution $\mathbf{Y}(t)$ of system (4).

Definition 1. Network (1) is said to achieve bounded synchronization to the convergence domain $\mathcal{M}$ if $\forall i, j = 1, \ldots, N$, $\mathbf{X}_{ij}(t)$ approaches to $\mathcal{M}$, i.e., $\lim_{t\to\infty} \mathrm{dist}(\mathbf{X}_{ij}(t), \mathcal{M}) = 0$, where $\mathbf{X}_{ij}(t) = \mathbf{x}_i(t) - \mathbf{x}_j(t)$, $\mathrm{dist}(\mathbf{x}^*, \mathcal{M})$ denotes the distance from a point $\mathbf{x}^*$ to a set $\mathcal{M}$, that is, the smallest distance from $\mathbf{x}^*$ to any point in $\mathcal{M}$.