Automatica 56 (2015) 86–92

Contents lists available at ScienceDirect

Automatica

journal homepage: www.elsevier.com/locate/automatica

Brief paper

Containment control for a social network with state-dependent connectivity☆

Zhen Kan[a], Justin R. Klotz[a], Eduardo L. Pasiliao Jr.[b], Warren E. Dixon[a]

[a] Department of Mechanical and Aerospace Engineering, University of Florida, Gainesville, USA

[b] Air Force Research Laboratory, Munitions Directorate, Eglin AFB, FL 32542, USA

ARTICLE INFO

Article history:
Received 7 December 2012
Received in revised form
5 December 2014
Accepted 12 March 2015
Available online 15 April 2015

Keywords:
Social network
Fractional-order dynamics
Containment control
Network connectivity

ABSTRACT

Social interactions influence our thoughts, opinions and actions. In this paper, social interactions are studied within a group of individuals composed of influential social leaders and followers. Each person is assumed to maintain a social state, which can be an emotional state or an opinion. Followers update their social states based on the states of local neighbors, while social leaders maintain a constant desired state. Social interactions are modeled as a general directed graph where each directed edge represents an influence from one person to another. Motivated by the non-local property of fractional-order systems, the social response of individuals in the network are modeled by fractional-order dynamics whose states depend on influences from local neighbors and past experiences. A decentralized influence method is then developed to maintain existing social influence between individuals (i.e., without isolating peers in the group) and to influence the social group to a common desired state (i.e., within a convex hull spanned by social leaders). Mittag-Leffler stability methods are used to prove the asymptotic convergence of the networked fractional-order system.

1. Introduction

Social interactions influence our emotions, opinions, and behaviors. Technological advances in social media provide more rapid, convenient, and widespread communication among individuals, which leads to a more dynamic interaction and influence. For example, recent riots (Bright, 2011) and ultimately revolution (Gustin, 2011), have been facilitated through social media technologies such as Facebook, Twitter, YouTube, and BlackBerry Messaging (BBM). Marketing agencies also have begun to take advantage of influence due to social media, especially through the internet. The company Razorfish, for example, works with online peer influencers to transform them into brand advocates through the execution of Social Influence Marketing (SIM) Strategy, which aims to influence marketing primarily through online, small groups, peer pressure, reciprocity or flattery (Singh, 2009).

Various dynamic models have been developed to study the individual's social behavior, such as the efforts to model the emotional response of different individuals (Ghosh, 2010; Sprott, 2004, 2005). In Sprott (2004), the time-variation of emotions between individuals involved in a romantic relationship is described by a dynamic model of love, and in Sprott (2005) a set of differential equations are developed to model the individual's happiness in response to exogenous influences. Fractional-order differential equations are a generalization of integer-order differential equations, and they exhibit a non-local integration property where the next state of a system not only depends upon its current state but also upon its historical states starting from the initial time (Monje, Chen, Vinagre, Xue, & Feliu, 2010). Motivated by this property, many researchers have explored the use of fractional-order systems as a model for various phenomena in natural and engineered systems. For instance, the works in Sprott (2004, 2005) were revisited in Ahmad and El-Khazali (2007) and Song, Xu, and Yang (2010), where the models of love and happiness were generalized to fractional-order dynamics by taking into account the fact that a person's emotional response is influenced by past experiences and memories. However, the models developed in Ahmad and El-Khazali (2007), Sprott (2004, 2005) and Song et al. (2010) only focus on an individual's emotional response, without considering the interaction with

☆ This research is supported in part by NSF award numbers 1161260, 1217908, and a contract with the AFRL Mathematical Modeling and Optimization Institute. Any opinions, findings and conclusions or recommendations expressed in this material are those of the authors and do not necessarily reflect the views of the sponsoring agency. The material in this paper was partially presented at the 2012 American Control Conference, June 27–29, 2012, Montreal, Canada and at the 2013 American Control Conference, June 17–19, 2013, Washington, DC, USA. This paper was recommended for publication in revised form by Associate Editor Zhihua Qu under the direction of Editor Andrew R. Teel.

E-mail addresses: kanzhen0322@ufl.edu (Z. Kan), jklotz@ufl.edu (J.R. Klotz), pasiliao@eglin.af.mil (E.L. Pasiliao Jr.), wdixon@ufl.edu (W.E. Dixon).

http://dx.doi.org/10.1016/j.automatica.2015.03.026
0005-1098/

social peers where rapid and widespread influences from social peers can prevail. Other results, such as Blondel, Hendrickx, and Tsitsiklis (2009) and Cucker and Smale (2007) and the reference therein, studied the interaction of social peers using an opinion dynamics model, and derived conditions under which consensus can be reached. However, agents in Blondel et al. (2009) and Cucker and Smale (2007) only update their opinions by averaging the neighboring agent opinions, without taking into account the influence of agents' past experience and memory on their decision making.

When making a decision or forming an opinion, individuals tend to communicate with parents, friends, or colleagues and take advice from social peers. Social connections such as friendship, kinship, and other relationships can influence the decisions they make. Some individuals (e.g., parents, teachers, mentors, and celebrities) may exhibit more powerful influences in others' decision making, and the underlying social network enables the influence to pass from influential individuals to receptive individuals. Containment control is a particular class of consensus problems (see Olfati-Saber, Fax, & Murray, 2007 and Ren, Beard, & Atkins, 2007 for a comprehensive literature review for consensus problems), in which follower agents are under the influence of leaders through local information exchange in a leader–follower network. In results such as Cao and Ren (2009), Cao, Ren, and Egerstedt (2012), Mei, Ren, and Ma (2012) and Notarstefano, Egerstedt, and Haque (2011), distributed containment control algorithms are developed for agents with integer-order dynamics where the group of followers is driven to a convex hull spanned by multiple leaders' states under an undirected, directed or switching topology. This paper examines how such methods can be leveraged to manipulate a social network. This work specifically aims to investigate how peer pressure from social leaders affects consensus beliefs (e.g., opinions, emotional states, purchasing decisions, political affiliation, etc.) within a social network, and how an interaction algorithm can be developed such that the group social behavior can be driven to a desired end (i.e., a convex hull spanned by the leaders' states).

By modeling human emotional response as a fractional-order system, the influence of a person's emotions within a social network is studied, and emotion synchronization for a group of individuals is achieved in our recent preliminary work (Kan, Klotz, Pasiliao, & Dixon, 2013; Kan, Shea, & Dixon, 2012). However, the emotion synchronization behavior in Kan, Shea et al. (2012) only considers an undirected network structure: the one-sided influence of social leaders is not considered. This work aims to investigate how the social beliefs (e.g., emotional response, opinions, etc.) of a group of individuals evolve under the influence of social leaders. Similar to Kan, Shea et al. (2012), the social group is modeled as a networked fractional-order system, where the social response of each individual is described by fractional-order dynamics whose states depend on influences from social peers, as well as past experiences. Since social leaders are considered, the undirected network topology in Kan, Shea et al. (2012) is extended to a directed graph, where the directed edges indicate the influence capability between two individuals (e.g., the leaders can influence the followers' state, but not vice versa). The goal in this work is to develop a decentralized influence algorithm where individuals within a social group update their beliefs by considering beliefs from social peers and the social group achieves a desired common belief (i.e., the social state of the group converges to a convex hull spanned by social leaders). Since an individual generally only considers others' beliefs as reasonable if their beliefs differ by less than a threshold, social difference is introduced to measure the closeness of the beliefs between individuals. In contrast to the constant weights considered in Cao and Ren (2009), Mei et al. (2012) and Notarstefano et al. (2011), the social difference is a time-varying weight which depends on individuals' current states. Moreover, instead of assuming network connectivity (i.e., there always exists a path of influence between any two agents) such as in Cao and Ren (2009), Mei et al. (2012) and Notarstefano et al. (2011), one main challenge here is to influence the followers' social states to a desired end by maintaining consistent interaction among social peers and influential leaders (i.e., individuals can always be influenced by social peers, instead of being isolated from the social group) within a time-varying graph. When modeled as a networked fractional-order system, the development of a containment algorithm can be more challenging compared to the integer-order dynamics in Cao and Ren (2009), Cao et al. (2012), Mei et al. (2012) and Notarstefano et al. (2011), which can be considered as a particular case of generalized fractional-order dynamics. The first apparent result that investigated the coordination of networked fractional systems is Cao, Li, Ren, and Chen (2010). However, only linear time-invariant systems are considered in Cao et al. (2010), where the interaction between agents is modeled as a link with a constant weight. Due to the time-varying weights considered here, previous stability analysis tools such as examining the eigenvalues of linear time-invariant fractional-order systems (cf. Cao et al., 2010, Chen, Ahn, & Podlubny, 2006 and Song et al., 2010) are not applicable to the time-varying networked fractional-order system in this work. To address these challenges, a decentralized influence function is developed to achieve containment control for the networked fractional-order systems while preserving continued social interaction among individuals. Asymptotic convergence of the social states to the convex hull spanned by leaders' states in the social network is then analyzed via LaSalle's invariance theorem (Khalil, 2002), convex properties (Boyd & Vandenberghe, 2004) and a Mittag-Leffler stability (Li, Chen, & Podlubny, 2009) approach.

2. Preliminaries

Consider a Fractional Order System (FOS)

$$_{t_0}D_t^{\alpha} x(t) = f(t, x) \tag{1}$$

with initial condition[1] $x(t_0)$, where $_{t_0}D_t^{\alpha}$ denotes the fractional derivative operator with order $\alpha \in (0, 1]$ on a time interval $[t_0, t]$, and $f(t, x)$ is piecewise continuous in t and locally Lipschitz in x. Similar to the exponential function used in solutions of integer-order differential equations, the Mittag-Leffler (M-L) function given by $E_{\alpha,\beta}(z) = \sum_{k=0}^{\infty} \frac{z^k}{\Gamma(k\alpha+\beta)}$, where $\alpha, \ \beta > 0$ and $z \in \mathbb{C}$, is frequently used in solutions of fractional-order systems (Monje et al., 2010). Particularly, when $\alpha = \beta = 1, \ E_{1,1}(z) = e^x$ is an exponential function. Stability of the solutions to (1) is defined by the M-L function as follows Li et al. (2009).

Definition 1 (*Li et al., 2009 (Mittag-Leffler Stability)*)**.** The solution of (1) is said to be Mittag-Leffler stable if $\|x(t)\| \leq \left\{m[x(t_0)] E_{\alpha,1}\left(-\lambda (t - t_0)^{\alpha}\right)\right\}^b$, where t_0 is the initial time, $\alpha \in (0, 1)$, $b > 0$, $\lambda > 0$, $m(0) = 0$, $m(x) \geq 0$, $m(x)$ is locally Lipschitz, and $E_{\alpha,1}$ is defined as $E_{\alpha,\beta}$ with $\beta = 1$.

Lyapunov's direct method is extended to fractional-order systems in the following lemma to determine Mittag-Leffler stability for the solutions of (1) in Li et al. (2009).

[1] The initial condition $x(t_0)$ is defined as a linear combination of internal states $z_k(t_0)$, $k = 1, \ldots, J$, where $z_k(t_0)$ contains all historical information of the system for $t < t_0$ based on the work in Trigeassou and Maamri (2011). The infinite state model approach to resolve the initialization in Trigeassou and Maamri (2011) is also used in the subsequent simulation section.