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Brief paper

Passivity-based synchronization of a class of complex dynamical networks with time-varying delay ☆

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ABSTRACT

This paper proposes a complex delayed dynamical network consisting of N linearly and diffusively coupled identical reaction–diffusion neural networks. By utilizing some inequality techniques, a sufficient condition ensuring the output strict passivity is derived for the proposed network model. Then, we reveal the relationship between output strict passivity and synchronization of the proposed network model. Moreover, based on the obtained passivity result and the relationship between output strict passivity and synchronization, a criterion for synchronization is established. Finally, a numerical example is provided to illustrate the correctness and effectiveness of the proposed results.

1. Introduction

Today various complex networks can be seen everywhere and are becoming an important part of our daily life. Some of the most well-known examples include food webs, communication networks, social networks, power grids, cellular networks, World Wide Web, metabolic systems, disease transmission networks, etc. Therefore, the topology and dynamical behavior of complex dynamical networks have been extensively studied by the researchers. In particular, the synchronization problem of complex dynamical networks has received much of the focus in recent years. So far, a great many important results on synchronization have been obtained for various complex networks (Arcak, 2011; Lü & Chen, 2005; Scardovi, Arcak, & Sontag, 2010; Slotine & Wang, 2005; Yao, Guan, & Hill, 2009; Yao, Wang, Guan, & Xu, 2009; Zhou, Lu, & Lü, 2006, 2008). In Slotine and Wang (2005), the authors studied the synchronization of a network with fixed and switching topologies by using partial contraction theory. In Arcak (2011), a sufficient condition was obtained to guarantee the synchronization of a compartmental ODE model by utilizing the properties of the Laplacian matrix and the Mean-Value theorem. However, in these existing works, the node state is only dependent on the time. But, in many circumstances, the node state is not only dependent on the time, but also intensively dependent on space variable.

As a special class of complex networks, arrays of coupled neural networks have attracted much attention in recent years. Especially, the synchronization problem of arrays of coupled neural networks has stirred much research interest due to its fruitful applications in various fields. In Hoppensteadt and Izhikevich (2000), the authors proposed an architecture of coupled neural networks to store and retrieve complex oscillatory patterns as synchronization states. In Zhang and He (1997), a secure communication system based on coupled cellular neural networks was presented. In addition, the study of synchronization of coupled neural networks is an important step for understanding brain science (Gray, 1994; Ukhtomsky, 1978). Therefore, it is interesting to investigate the synchronization of coupled neural networks. In Tang and Fang (2009), the authors considered a general model of an array of N linearly coupled delayed neural networks with

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Markovian jumping hybrid coupling, which is composed of constant coupling, discrete and distributed time-varying delay coupling. In Li, Song, and Fei (2010), an array of coupled discrete-time Cohen–Grossberg neural networks with time-varying delay was discussed. However, in these existing works (Li et al., 2010; Tang & Fang, 2009), the diffusion effects have not been considered. Strictly speaking, diffusion effects cannot be avoided in neural networks when electrons are moving in asymmetric electromagnetic fields, thus we must consider the diffusion effects in neural networks. To our knowledge, very few researchers have investigated the synchronization in coupled reaction–diffusion neural networks (Liu, 2010; Wang, Teng, & Jiang, 2012). In Liu (2010), the author investigated a class of linearly coupled reaction–diffusion neural networks with unbounded time delays. In Wang, Teng et al. (2012), the adaptive synchronization in an array of linearly coupled neural networks with reaction–diffusion terms and time delays was studied.

Recently, passivity theory has also received a great deal of attention, and many results on this topic have been reported. The passivity theory was firstly proposed in the circuit analysis (Bevelevich, 1968), and since then has found successful applications in diverse areas such as stability, complexity, signal processing, chaos control and synchronization, fuzzy control, group coordination, power control, flow control, energy management, and so on (Arcak, 2007; Hill & Moylan, 1977; Wu, 2001; Xie, Fu, & Li, 1998). Although research on passivity has attracted so much attention, little of that had been devoted to the passivity properties of the spatially and temporally complex dynamical networks until Wang, Wu and Guo (Wang, Wu, & Guo, 2011) obtained the conditions for passivity of reaction–diffusion neural networks. To the best of our knowledge, the passivity of arrays of coupled reaction–diffusion neural networks has not yet been considered. Therefore, it is important and interesting to study the passivity of coupled reaction–diffusion neural networks. On the other hand, the passivity theory has long been a nice tool for analyzing the synchronization of the complex networks. But in most existing works, it is assumed that the node state is only dependent on the time. Therefore, it is essential to investigate the relationship between passivity and synchronization of the coupled reaction–diffusion neural networks.

The objective of this paper is to study the synchronization problem of arrays of coupled reaction–diffusion neural networks by using the passivity theory. The main contributions of this paper are as follows. First, we establish a criterion for the output strict passivity by utilizing some inequality techniques. Second, we reveal the relationship between output strict passivity and synchronization of the proposed network model. Third, by employing the obtained passivity result and the relationship between output strict passivity and synchronization, a sufficient condition for synchronization of the complex dynamical network is derived.

The rest of this paper is organized as follows. In Section 2, our mathematical model of complex network is presented and some preliminaries are given. The main results of this paper are given in Section 3. In Section 4, a numerical example is provided to illustrate the effectiveness of the theoretical results. Finally, Section 5 concludes the investigation.

2. Network model and preliminaries

Let $\mathbb{R} = (-\infty, +\infty)$, $\mathbb{R}^+ = [0, +\infty)$, $\mathbb{R}^n$ be the n-dimensional Euclidean space and $\mathbb{R}^{n\times m}$ be the space of $n \times m$ real matrices. $P \in \mathbb{R}^{n\times n} \geqslant 0$ ($P \in \mathbb{R}^{n\times n} \leqslant 0$) means that matrix P is symmetric and semi-positive (semi-negative) definite. $P \in \mathbb{R}^{n\times n} > 0$ ($P \in \mathbb{R}^{n\times n} < 0$) means that matrix P is symmetric and positive (negative) definite. I_n denotes the $n \times n$ real identity matrix. B^T denotes the transpose of matrix B. $\otimes$ denotes the Kronecker product of two matrices. $\lambda_m(\cdot)$ and $\lambda_M(\cdot)$ denote the minimum and the maximum eigenvalue of the corresponding matrix, respectively. $\Omega = \{x = (x_1, x_2, \ldots, x_q)^T \mid |x_k| < l_k, k = 1, 2, \ldots, q\}$ is an open bounded domain in $\mathbb{R}^q$ with smooth boundary $\partial\Omega$, $\overline{\Omega} = \Omega \cup \partial\Omega$, and mes$\Omega$ denotes the measure of Ω. For any $e(x, t) = (e_1(x, t), e_2(x, t), \ldots, e_n(x, t))^T \in \mathbb{R}^n$, $(x, t) \in \Omega \times \mathbb{R}$, $\|e(\cdot, t)\|_2$ denotes

$$\|e(\cdot, t)\|_2 = \left(\int_\Omega \sum_{i=1}^n e_i^2(x, t)dx \right)^{\frac{1}{2}}.$$

In addition, we define $\|e(\cdot, t)\|_\tau = \sup_{-\tau\leqslant\theta\leqslant 0} \|e(\cdot, t + \theta)\|_2$.

In this paper, we consider a complex dynamical network consisting of N identical reaction–diffusion neural networks. To facilitate the readers, the complex network model is presented in a step-by-step format.

A single reaction–diffusion neural network with Dirichlet boundary conditions is described by the following partial differential equations (PDEs):

$$\frac{\partial w_i(x, t)}{\partial t} = d_i \triangle w_i(x, t) - a_i w_i(x, t) + J_i + \sum_{j=1}^n b_{ij} f_j(w_j(x, t)) \quad (1)$$

where $i = 1, 2, \ldots, n$, n is the number of neurons in the network; $x = (x_1, x_2, \ldots, x_q)^T \in \Omega \subset \mathbb{R}^q$; $w_i(x, t) \in \mathbb{R}$ is the state of the ith neuron at time t and in space x; $\triangle = \sum_{k=1}^q \frac{\partial^2}{\partial x_k^2}$ is the Laplace diffusion operator on Ω; $d_i > 0$ represents the transmission diffusion coefficient along the ith neuron; $f_j(\cdot)$ denotes the activation function of the jth neuron; $a_i > 0$ represents the rate with which the ith neuron will reset its potential to the resting state when disconnected from the network and external input; b_{ij} denotes the strength of the jth neuron on the ith neuron; J_i is a constant external input.

The initial value and boundary value conditions associated with system (1) are given in the form

$$w_i(x, 0) = \phi_i(x), \quad x \in \Omega, \quad (2)$$

$$w_i(x, t) = 0, \quad (x, t) \in \partial\Omega \times [0, +\infty) \quad (3)$$

where $\phi_i(x)(i = 1, 2, \ldots, n)$ is bounded and continuous on Ω.

Throughout this paper, the function $f_j(\cdot)(j = 1, 2, \ldots, n)$ satisfies the Lipschitz condition, that is, there exists positive constant ρ_j such that

$$|f_j(\xi_1) - f_j(\xi_2)| \leqslant \rho_j|\xi_1 - \xi_2|$$

for any $\xi_1, \xi_2 \in \mathbb{R}$, where $|\cdot|$ is the Euclidean norm.

We can rewrite system (1) in a compact form as follows:

$$\frac{\partial w(x, t)}{\partial t} = D\triangle w(x, t) - Aw(x, t) + J + Bf(w(x, t)) \quad (4)$$

where $D = \text{diag}(d_1, d_2, \ldots, d_n)$, $B = (b_{ij})_{n\times n}$, $J = (J_1, J_2, \ldots, J_n)^T$, $A = \text{diag}(a_1, a_2, \ldots, a_n)$, $f(w(x, t)) = (f_1(w_1(x, t)), f_2(w_2(x, t)), \ldots, f_n(w_n(x, t)))^T$, $w(x, t) = (w_1(x, t), w_2(x, t), \ldots, w_n(x, t))^T$.

N mutually coupled reaction–diffusion neural networks (4) can result in a complex network, which is described by

$$\frac{\partial z_i(x, t)}{\partial t} = D\triangle z_i(x, t) - Az_i(x, t) + Bf(z_i(x, t)) + c_1 \sum_{j=1}^N G_{ij}^1 \Gamma_1 z_j(x, t) + J + c_2 \sum_{j=1}^N G_{ij}^2 \Gamma_2 z_j(x, t - \tau(t)) + u_i(x, t) \quad (5)$$