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Survey paper

Multivariable adaptive control: A survey[☆]

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ABSTRACT

Adaptive control is a control methodology capable of dealing with uncertain systems to ensure desired control performance. This paper provides an overview of some fundamental theoretical aspects and technical issues of multivariable adaptive control, and a thorough presentation of various adaptive control schemes for multi-input–multi-output systems, literature reviews on adaptive control foundations and multivariable adaptive control methods, and related technical problems. It covers some basic concepts and issues such as certainty equivalence, stability, tracking, robustness, and parameter convergence. It discusses some of the most important topics of adaptive control: plant uncertainty parametrization, stable controller adaptation, and design conditions for different adaptive control schemes. The paper also presents a detailed study of well-developed multivariable model reference adaptive control theory and design techniques. It provides an introduction to multivariable adaptive pole placement and adaptive nonlinear control, and it concludes by identifying some open research problems.

1. Introduction

As a popular control methodology of increasing interest for applications in engineering and science fields, adaptive control has its unique capabilities to accommodate system parametric, structural, and environmental uncertainties caused by payload variations or system aging, component failures, and external disturbances. Adaptive control systems, under some generic design conditions, are capable of tolerating large parametric, structural and parametrizable disturbance uncertainties, to ensure desired system asymptotic tracking performance, in addition to system stability. Such asymptotic tracking performance is crucial for many performance-critical system applications such as aircraft control and cyber-physical systems, and is desirable for resilient control systems whose performance is required to be recoverable under system uncertainties and faults.

Multivariable adaptive control (adaptive control of multivariable or multi-input, multi-output systems) is a research area presenting both theoretical challenges and practical significance, with unique technical issues. Many practical systems have multiple inputs and multiple outputs, especially, those in the emerging technology applications. A key technical issue in multivariable adaptive control is how to deal with dynamic interactions between system inputs and outputs, in addition to other issues in dealing with parametric, structural, and environmental uncertainties as in the single-input, single-output systems. In this paper, we conduct a comprehensive study on some notable theoretical developments, key technical issues and potential research problems of multivariable adaptive control, and provide an extensive literature survey.

The study will start next in Section 2 with an overview of some basic topics of adaptive control. Section 3 provides a review of some representative adaptive control literature, focused on the foundations of adaptive control and on the developments of multivariable adaptive control. Section 4 presents a detailed demonstration of multivariable adaptive control of linear systems. It covers

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the controller parametrization, error equation, adaptive law, stability and robustness, for different types of adaptive control systems: state feedback for state tracking, state feedback for output tracking, output feedback for output tracking, discrete-time adaptive control systems, adaptive backstepping control, adaptive pole placement control, robustness and performance, etc. Section 5 addresses some basic issues of multivariable adaptive control of nonlinear systems, with parametrizable nonlinearities and with nonlinearity approximations. Section 6 provides a perspective on multivariable adaptive control and discusses some possible open areas of research.

The unified theme of adaptive control is the use of a certainty equivalence principle, based on two technical foundations: maximal plant uncertainty parametrization and stable controller parameter adaptation, which will be demonstrated in Sections 4 and 5.

There are other important adaptive control topics such as adaptive predictive control, adaptive learning control, adaptive variable structure control, adaptive control of robotic systems, adaptive systems with delays, stochastic adaptive control, and adaptive control of hybrid systems, which are not covered in this paper.

Adaptive control is a very broad field. This paper can only provide a limited study of it and discuss a small portion of its literature. There are many other interesting, important and respectable contributions in the literature, which could not be reported here.

Throughout this paper, we will use the notation: $y(t) = G(s)[u](t)$, to denote the output $y(t)$ of a linear time-invariant (LTI) system represented by a transfer matrix $G(s)$ with input $u(t)$. This notation is simple (to combine both time and frequency-domain signal operations) and useful (to avoid an inconvenient convolution expression) for adaptive control system presentation. For example, a common error system model is with $e(t)$ as output, $W_m(s)$ as system and $K_p(\Theta(t) - \Theta^*)^T\omega(t)$ as input (for some matrices K_p, $\Theta(t)$, Θ^* and vector $\omega(t)$), which can be expressed as: $e(t) = W_m(s)[K_p(\Theta - \Theta^*)^T\omega](t)$.

2. Adaptive control essentials

In this section, we address some fundamental topics of adaptive control including control performance in terms of system modeling, and control design based on the certainty equivalence principle, and some related technical issues such as convergence and robustness.

2.1. Adaptive control performance

We first illustrate that desired adaptive control system performance is ensured under certain modeling conditions. Performance robustness with respect to modeling errors is an important issue, and proper uncertainty parametrization is a key to performance improvement.

Basic adaptive control problems. For a system $\dot{x} = Ax + Bu,\ y = Cx$, or $\dot{x} = f(x) + g(x)u$, $y = h(x)$, with *unknown* parameters, what adaptive control can achieve are: system *stability* (signal boundedness), and *asymptotic tracking*, despite system parameter uncertainties. To fulfill these tasks, an adaptive controller automatically or adaptively adjusts its parameters based on performance errors, for which the controller structure choice and parameter adaptation design are most crucial.

Ideal performance. The distinctive feature of adaptive feedback control designs is that they do not need the knowledge of the parameters of the controlled system (plant) and can still achieve desired closed-loop system stability and asymptotic output or state tracking. A basic condition for such ideal adaptive control performance is that knowledge of the plant's dynamic order (or its upper bound, for model reference adaptive control) is used in the control law design. This means that all plant dynamics are modeled in the plant model. Another basic condition is that the plant is disturbance-free.

Such conditions, in the single-input–single-output (SISO) LTI case, require that (i) the plant is in a disturbance-free differential equation form: $P(s)[y](t) = Z(s)[u](t)$, and (ii) the controller order is not less than the order of $P(s)$, in order to ensure $\lim_{t\to\infty}(y(t) - y_m(t)) = 0$ for some pre-specified reference output signal $y_m(t)$.

Performance robustness. In the presence of modeling errors such as plant multiplicative unmodeled dynamics $\Delta_m(s)$ and additive unmodeled dynamics $\Delta_a(s)$ and disturbance $d(t)$, the plant model is

$$y(t) = (G(s)(1 + \Delta_m(s)) + \Delta_a(s))[u](t) + d(t), \tag{1}$$

for which an adaptive controller originally designed for $G(s) = Z(s)/P(s)$ may not ensure a stable control system. This is the well-known robustness issue of adaptive control, which has driven the developments and maturing of adaptive control theory and techniques. There are several robust adaptive control techniques using robust adaptive laws, robust control signals (such as bounding, switching or sliding-mode signals), or additional excitation signals. The robust adaptive control theory and design techniques have been well-developed, for (i) the closed-loop system stability in the presence of bounded disturbances $d(t)$ and some moderate unmodeled dynamics $\Delta_m(s)$ and $\Delta_a(s)$, despite the large parameter uncertainties of $G(s)$, and (ii) the tracking error which is bounded by the magnitudes of $d(t)$ and $\Delta_m(s)$ and $\Delta_a(s)$ operating on $u(t)$, in an average sense (Ioannou & Sun, 1996; Narendra & Annaswamy, 1989). For a survey of robust adaptive control theory and techniques, see Ortega and Tang (1989).

2.2. Principles of adaptive control

Realization of adaptation in the form of control theory is through an adaptive law (in terms of certain differential equations) which automatically adjusts the parameters of a feedback controller for a dynamical system with uncertain parameters, and has desired stability behavior.

The most important and unifying principle of adaptive control is the *certainty equivalence principle*. Two most important technical foundations to implement the certainty equivalence principle are *maximal plant uncertainty parametrization* and *stable controller parameter adaptation*, together making adaptive control a powerful and desirable methodology to deal with uncertainties.

The certainty equivalence principle. Adaptive control designs are based on the so-called *certainty equivalence principle*: for a plant with uncertain parameters, the adaptive parameter estimates are used in the feedback control design as if they are the true parameters.

An immediate question is: do parameter estimates in adaptive control converge to their true values? In adaptive estimation, the parameter estimates converge to their true values under some key conditions: (i) the plant model does not have zero-pole cancellation, and the plant estimate model has the same plant order, and (ii) the adaptive system is persistently exciting, ensured by condition (i) and by a sufficiently rich input.

For adaptive control, conditions (i) and (ii) may not be satisfied (as the modeling of a controlled but uncertain plant may not be minimal and the control system behavior may need to follow certain trajectory which may not be rich). However the certainty equivalence principle has been verified to work in adaptive control. The simple reason for this important accomplishment is that for adaptive control, the goal of stabilization and tracking control is achieved when the system only needs to operate in a subspace of signals of interest. Then, the control system has certain