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Generalized mode acceleration and modal truncation augmentation methods for the harmonic response analysis of nonviscously damped systems

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The modal truncation problem is frequently encountered in nonviscously (viscoelastically) damped systems since only the modes of interest are usually considered in the dynamic analysis of engineering problems. This study aims at accurately calculating the steady-state responses of nonviscously damped systems by only considering the modes of interest. Based on the Neumann expansion theorem and the frequency shifting technique, a property obtained from the first-order terms of the Neumann expansion of the frequency response function (FRF) matrix of nonviscously damped systems is given. However, this procedure cannot be extended to consider the further higher-order terms. It means a truncation expansion problem exists for nonviscously damped systems. By considering the first-order terms of the Neumann expansion, a generalized mode acceleration method (GMAM) is presented to handle the modal truncation problem. The GMAM can overcome the singular problem of the stiffness matrix. The modal truncation augmentation method (MTAM) is also presented to handle the modal truncation problem by making the equilibrium equations into a subspace equation spanned in terms of the columns of a projection basis given in the GMAM. Several conclusions concerning the implementation of the presented methods are formulated on the basis of the results of three examples.

1. Introduction

Harmonic response analysis is used to analyze the systems subject to steady-state oscillatory excitation, which may be caused by reciprocating or rotating machine parts (e.g., motors, fans, compressors, and forging hammers). Steady-state response analysis plays a very important role in many areas, including model updating, vibration and noise control, model identification and structural damage detection. The most common method to calculate the response may be the mode mode superposition method, which calculates the response by means of the superposition of free vibration modes of the systems. The mode superposition method requires all the frequencies and mode shapes to perform an exact response. Increasing the number of degrees of freedom used in dynamic analysis, the solution of all the frequencies and mode shapes is very time-consuming or even impossible. In practice, only the modes of interest are usually considered in the dynamic analysis of

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engineering problems. The response may be calculated by the superposition of free vibration modes of interest. This method is referred to as the mode displacement method (MDM). Since the contribution of unavailable modes is neglected, the quality of the response obtained using the MDM may be adversely affected. Many correction methods are presented to reduce the modal truncation error.

Mode acceleration method (MAM) [1,2], which was presented by combining the mode superposition of the available modes and the particular solution of the dynamic equation of motion when the excitation frequency equals zero. This approach is a static correction method since it only gives an exact result at zero frequency. The mode acceleration method can give an improvement on the response analysis, especially for the lower frequency range. Sometimes the actual remaining error of the MAM may be also big. High accurate mode superposition methods [3–5] were developed to reduce the actual remaining error by making use of the explicit series expansion of the contribution of the unavailable modes. The explicit series expansion is established by expressing it as a sum of the available modes and system matrices. However, these methods [3–5] are derived in terms of normal modes, namely, they are restricted in undamped or classically damped systems. In general, classical damping means that energy dissipation is almost uniformly distributed throughout the mechanical system. Also, a rigorous condition was proved by Caughey and O'Kelly [6]. There is, of course, no reason why the rigorous condition expressed as a mathematical formula must be satisfied. In practical, structural systems with two or more parts with significantly different levels of energy dissipation are frequently encountered. It was shown using experimental data [7] that no physical system is strictly classically damped. Recently, Li et al. [8] established the explicit series expansion satisfied by complex modes obtained from non-classically damped systems, and presented a high accurate modal superposition method to handle the modal truncation problem encountered in non-classically damped systems. Some other techniques are introduced to reduce the modal truncation error (see the recent review article [9] for details). However, these methods mentioned above are only restricted to the case of undamped or viscously damped systems.

Increasing the use of nonviscous (viscoelastic) damping to model and analyze mechanical systems (such as nano-composites and their applications in new generation of aircrafts, large wind turbines etc., active control involving piezoelectric and electromagnetic principles), has led to the need for calculating the responses of nonviscously damped systems in an accurate manner [10]. Often these nonviscous damping models are frequency-dependent. The modal truncation problem is more frequently encountered in nonviscously damped systems due to the fact that these systems product nonlinear eigenproblems, which are difficult or even impossible to be found accurately even for small-scaled problems. Recently, Li et al. [11] developed two correction modal methods for nonviscously damped systems by using the first one or two terms of the Neumann expansion of the contribution of the unavailable modes. However, these methods [11] cannot consider the further high-order terms, namely, a truncation expansion problem exists in these methods. The posteriori method [12] was therefore presented to improve the accuracy of these methods by making the equilibrium equations of motion into a subspace equation spanned in terms of the columns of a projection basis given in Ref. [11]. The focus of these correction modal methods [11,12] for the harmonic analysis of nonviscously damped systems was on taking into account the contribution of higher (unavailable) modes in terms of the lower modes and system matrices. However, these methods [11,12] require that the inverse of stiffness matrix must be available. It means that the stiffness matrix should be non-singular. As we know, sometimes the stiffness matrix is singular, e.g., in the case of free-free boundary conditions. The *singular problem* of the stiffness matrix poses some serious limitations in the experimental and computational calculation of responses. In addition, these methods [11,12] assume that only the higher-modes are unavailable. In many cases, only the modes of interest are calculated and used for dynamic analysis. For example, when the excitation frequencies lie in the frequency range, which are away from the lowest or base frequency. These problems motivate the development of more generalized methods to handle the modal truncation problem encountered in the harmonic responses of nonviscously damped systems.

This study aims at accurately calculating the harmonic responses of nonviscously damped systems in terms of the modes of interest. Based on the Neumann expansion theorem and the frequency shifting technique, a property obtained from the first-order terms of the Neumann expansion of the frequency response function (FRF) matrix of nonviscously damped systems is given. However, this procedure cannot be extended to consider the further higher-order terms. It means a *truncation expansion problem* exists for nonviscously damped systems. By considering the first-order terms of the Neumann expansion, two modal correction methods are presented to handle the modal truncation problem of nonviscously damped systems. These two methods can overcome the singular problem of stiffness matrix and only involve the modes of interest. Three case studies are provided to illustrate the effectiveness of the derived results. It is shown that the error of the MDM may be unexpected and the presented methods can reduce the modal truncation error.

2. Theoretical background

The equations of motion of a linear nonviscously damped system with zero initial condition can be expressed as

$$M\ddot{\mathbf{u}}(t)+\mathbf{K}_V\int_0^t g(t-\tau)\frac{\partial\mathbf{u}(\tau)}{\partial\tau}\mathrm{d}\tau+\mathbf{K}\mathbf{u}(t)=\mathbf{f}(t) \tag{1}$$

where $\mathbf{M}$ and $\mathbf{K}\in\mathbb{R}^{N\times N}$ are, respectively, the mass and stiffness matrices, $\mathbf{f}(t)$ is the forcing vector. $g(t)$ is the kernel function, $\mathbf{u}(t)$ is the displacement vector, $t\in\mathbb{R}^+$ denotes time and over-dot denotes time derivative. The procedure of obtaining the stiffness matrix $\mathbf{K}$ can be seen in [11,13]. Here $\mathbf{K}_V$, which is the damping coefficient matrix, is used to cover the distribution

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