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Investigation of a jointed friction oscillator using the Multiharmonic Balance Method

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ABSTRACT

This paper covers the investigation of a nonlinear jointed structure in the frequency domain. Due to frictional nonlinearities the behavior of that system is approximated using the Multiharmonic Balance Method (MHBM). Two models of different complexity are presented. The first is a simple three degree-of-freedom model which is valid because of the special design of the structure. This model enables a very fast and accurate prediction of the stationary behavior of the real system. In order to attain a more general way of modeling structures including joints, the second model is a Finite Element (FE) model. For the discretization of the contact plane, "Zero Thickness" (ZT) elements are implemented. These elements allow the application of nearly arbitrary constitutive laws for describing the dynamic joint behavior. Here a coupled three dimensional contact law including dry friction effects is applied and the needed partial derivatives for the MHBM procedure are given analytically. Using measurements from the real structure and performing a model updating process, the parameters of the two presented models are estimated. The calculation results are compared to measurements in the frequency as well as in the time domain.

1. Introduction

Joints play an important role in most mechanical applications and the prediction of the dynamical behavior of jointed structures is a challenging task in structural dynamics. Their inclusion leads to, for example, shifts of the resonance frequencies or the maximal response amplitude within a resonance due to damping effects. These effects are known as "structural damping" [24,10]. They are crucial for structures, where the internal "material damping" either is quite low, as is the case for typical applications in mechanical engineering such as devices made of metal, and may even lead to instabilities in rotating machineries [7]. The effects, which can be summed up to the abstract term of structural damping, can often be traced back to the appearance of dry friction between the contacting parts. Friction occurs, on the microscopic scale, because of the sliding and the deformation of asperities, when the contacting bodies move relative to each other [18,39]. Since it is not practicable to take into account all microscopic details of contact surfaces for simulations of real mechanical devices, it is often aimed to find an analogous model on the macroscopic scale. This model represents a friction law, in terms of a constitutive contact law, by exploiting the knowledge following from classical contact mechanics [23,21] or being observed in measurements [27]. On this way a significant number of different friction models have been established [8,2], where particularly those based on rheological models have a broad range of application. Especially Masing and Iwan models [19,9],

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which are based on the parallel and serial arrangement of Jenkin elements [20] respectively, have to be pointed out as they are often used for representing the behavior of bolted joints [26,34,31]. A good overview of further literature on the framework of joints in general can be found in reference [3].

The typical form of the equations of motion for a system containing a (friction type) nonlinearity reads as

$$\boldsymbol{M}\ddot{\boldsymbol{u}}(t)+\boldsymbol{D}\dot{\boldsymbol{u}}(t)+\boldsymbol{K}\boldsymbol{u}(t)+\boldsymbol{f}_{nl}(\boldsymbol{u},\dot{\boldsymbol{u}},t)=\boldsymbol{f}_e(t), \tag{1}$$

containing not only the mass, damping and stiffness matrices $\boldsymbol{M}$, $\boldsymbol{D}$, $\boldsymbol{K}$, respectively, as well as the excitation forces $\boldsymbol{f}_e$, but additionally a vector of inner forces $\boldsymbol{f}_{nl}$ which may depend nonlinearly on the time, on the displacement vector $\boldsymbol{u}$ and its derivative $\dot{\boldsymbol{u}}$ with respect to time. In this contribution, the focus is on investigating the behavior of a jointed structure over a broad range of excitation frequencies within the frequency domain corresponding to the stationary behavior in the time domain. Due to the nonlinear forces $\boldsymbol{f}_{nl}$, it is not possible to perform an analytical transformation of the system equations into the frequency domain directly. Instead, a possible approach to approximate the nonlinear term is the usage of the Harmonic Balance Method (HBM) [40,9,13], which was originally proposed by Kryloff and Bogoliuboff [22]. In the framework of the HBM it is assumed that a harmonic excitation of the system leads to a harmonic response. A generalization or extension to periodic excitation and response functions is the Multiharmonic Balance Method (MHBM) which found its most popular application in the dynamic simulation of turbine bladings [29,33,35]. In the framework of this paper, the MHBM is realized using the Alternating Frequency Time Domain Method (AFT) [4,30,5]. The calculation procedure is applied to a friction oscillator, which is modeled firstly as a 3-DOF oscillator with a one dimensional contact law and secondly via a Finite Element (FE) model with a corresponding three dimensional contact law. To the authors knowledge, it is a novel approach, that a coupled three dimensional constitutive law accounting for dry friction is applied within the MHBM by considering an analytical formulation of the needed Jacobian.

2. Multiharmonic Balance Method

This section is provided to give a brief introduction to the MHBM and an overview of the chosen nomenclature. Within the scope of the MHBM the excitation forces $\boldsymbol{f}_e(t)$ as well as the response displacements $\boldsymbol{u}(t)$ of a system may be periodic in nature. They are approximated by the ansatz of a truncated Fourier series

$$\boldsymbol{f}_e(t)\approx\boldsymbol{F}_{e,(0)}+\sum_{k=1}^{n_h}(\tilde{\boldsymbol{F}}_{e,(k)}e^{i\cdot k\omega t}+\tilde{\boldsymbol{F}}^{*}_{e,(k)}e^{-i\cdot k\omega t}) \tag{2}$$

and

$$\boldsymbol{u}(t)\approx\boldsymbol{U}_{(0)}+\sum_{k=1}^{n_h}(\tilde{\boldsymbol{U}}_{(k)}e^{i\cdot k\omega t}+\tilde{\boldsymbol{U}}^{*}_{(k)}e^{-i\cdot k\omega t}), \tag{3}$$

which resolves n_h harmonic parts and neglects all higher harmonics. In these formulas the subscript numbers in brackets show the respective harmonic part, the $\sim$ indicates that the corresponding values are complex and $*$ stands for the conjugate complex.

The same ansatz is used for the nonlinear forces $\boldsymbol{f}_{nl}$ as well. If the excitation forces are harmonic or have a limited number of harmonics less than n_h, Eq. (2) can also be exactly valid. The conjugate complex parts can be dropped since they contain no additional information. So the equations of motion can be written as

$$\sum_{k=0}^{n_h}((-k^2\omega^2\boldsymbol{M}+i\cdot k\omega\boldsymbol{D}+\boldsymbol{K})\tilde{\boldsymbol{U}}_{(k)}e^{i\cdot k\omega t})+\sum_{k=0}^{n_h}(\tilde{\boldsymbol{F}}_{nl,(k)}e^{i\cdot k\omega t})=\sum_{k=0}^{n_h}(\tilde{\boldsymbol{F}}_{e,(k)}e^{i\cdot k\omega t}). \tag{4}$$

In order to solve this complex nonlinear system of equations it is transformed into a real valued form by splitting it into its real and imaginary part. Additionally, the equations are split into their different harmonic components. This increases the dimension of the problem by a factor of $n_{dim}=1+2\cdot n_h$. As an example, the real valued representation of the vector of response displacements in the frequency domain will read as

$$\boldsymbol{U}=[\boldsymbol{U}_{(0)}^T,\ \Re\{\tilde{\boldsymbol{U}}_{(1)}\}^T,\ \Re\{\tilde{\boldsymbol{U}}_{(2)}\}^T,\ \ldots,\ \Re\{\tilde{\boldsymbol{U}}_{(n_h)}\}^T,\ \Im\{\tilde{\boldsymbol{U}}_{(1)}\}^T,\ \Im\{\tilde{\boldsymbol{U}}_{(2)}\}^T,\ \ldots,\ \Im\{\tilde{\boldsymbol{U}}_{(n_h)}\}^T]^T. \tag{5}$$

Applying this to the complete system (4) the real valued harmonic component representation is

$$\boldsymbol{S}(\omega)\boldsymbol{U}(\omega)+\boldsymbol{F}_{nl}(\boldsymbol{U}(\omega))=\boldsymbol{F}_e(\omega), \tag{6}$$

where $\boldsymbol{S}$ denotes the diagonal block matrix of the real valued dynamic stiffness matrices

$$\boldsymbol{S}(\omega)=\begin{bmatrix}\boldsymbol{K} & \boldsymbol{0} & \boldsymbol{0}\\ \boldsymbol{0} & \mathrm{diag}(-k^2\omega^2\boldsymbol{M}+\boldsymbol{K}) & \mathrm{diag}(-k\omega\boldsymbol{D})\\ \boldsymbol{0} & \mathrm{diag}(k\omega\boldsymbol{D}) & \mathrm{diag}(-k^2\omega^2\boldsymbol{M}+\boldsymbol{K})\end{bmatrix}. \tag{7}$$

When looking again at the classical HBM at this point the nonlinear forces are derived by analytically calculating the Fourier coefficients. However, because of the coupling of all the harmonic components with each other, this step cannot practically be done when dealing with more than one harmonic as it is the case for the MHBM [30]. Therefore the Alternating Frequency Time Domain Method (AFT) [4] is applied here. In the framework of the AFT method, the complex valued