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Finite element model updating using the shadow hybrid Monte Carlo technique

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ABSTRACT

Recent research in the field of finite element model updating (FEM) advocates the adoption of Bayesian analysis techniques to dealing with the uncertainties associated with these models. However, Bayesian formulations require the evaluation of the Posterior Distribution Function which may not be available in analytical form. This is the case in FEM updating. In such cases sampling methods can provide good approximations of the Posterior distribution when implemented in the Bayesian context. Markov Chain Monte Carlo (MCMC) algorithms are the most popular sampling tools used to sample probability distributions. However, the efficiency of these algorithms is affected by the complexity of the systems (the size of the parameter space). The Hybrid Monte Carlo (HMC) offers a very important MCMC approach to dealing with higher-dimensional complex problems. The HMC uses the molecular dynamics (MD) steps as the global Monte Carlo (MC) moves to reach areas of high probability where the gradient of the log-density of the Posterior acts as a guide during the search process. However, the acceptance rate of HMC is sensitive to the system size as well as the time step used to evaluate the MD trajectory. To overcome this limitation we propose the use of the Shadow Hybrid Monte Carlo (SHMC) algorithm. The SHMC algorithm is a modified version of the Hybrid Monte Carlo (HMC) and designed to improve sampling for large-system sizes and time steps. This is done by sampling from a modified Hamiltonian function instead of the normal Hamiltonian function. In this paper, the efficiency and accuracy of the SHMC method is tested on the updating of two real structures; an unsymmetrical H-shaped beam structure and a GARTEUR SM-AG19 structure and is compared to the application of the HMC algorithm on the same structures.

1. Introduction

Finite element models (FEMs) are well-known numerical methods used to provide approximate solutions for complex engineering problems [1,2]. In mechanical engineering the FEM method is often used for computing displacements, stresses and strains in structures under a set of loads. However, FEM results degrade with an increase in problem complexity. The result is that the FEM results for the complex system differ from those obtained from experiments [3,4]. These differences can mainly be attributed to modeling errors, especially the uncertainties associated with modeling some structural feature

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(and properties as per manufacturing process) and or modeling the possible dynamics of those features. Therefore, the initial FE model needs to be updated to match the measured data. There are two main classes of model updating: direct method and the indirect (iterative) methods. The direct methods are computationally efficient approaches that update the FEM matrix components in one step [5,6]. However, high quality measurements as well as accurate FE models are required for this updating process. Using direct methods often results in unrealistic updating parameter values hence these parameters lose their physical meaning. On the other hand iterative FEM methods use cost functions to iteratively reduce the error between the experimental and analytical results. This is done by modifying (updating) the uncertain parameters in the modeled system [7]. The most common iterative approaches are the sensitivity-based updating methods see Link [8] for a review. In the last decade, the nature-inspired optimization algorithms have been widely applied in the area of model updating. These algorithms represent a class of probabilistic search algorithms that cope well with non-convex optimization problems. In these algorithms, controlled random steps are used during the searching procedure where the new search position is somehow based on the previous one. Levin et al. [9] applied the simulated annealing (SA) method and the genetic algorithm (GA) to update simulated and experimental systems while Marwala [4] used a Particle Swarm Optimization (PSO) algorithm to update a H-Beam structure.

Unfortunately, most of deterministic algorithms based on optimization do not quantify the involved uncertainties in structures tests. In the case of uncertain systems, the classical deterministic algorithms based on optimization do not quantify the involved uncertainties and this may degrade the accuracy of the obtained results. On the other hand, uncertainty quantification methods are designed to quantify uncertainties in engineering problems [4,7,10]. There are two main classes of uncertainty quantification methods: Probabilistic methods and non-probabilistic methods. Probabilistic methods such as Bayesian methods, perturbation methods are based on probability theory in which the uncertain parameters are modeled as random parameters. The interval arithmetic and the fuzzy methods belong to the non-probabilistic class [7].

In recent years Bayesian model updating techniques have shown promising results in systems identification type problems–of which FEM updating is one [4,11–13]. The Bayesian approach possesses the ability to characterize and quantify the uncertainties of a modeled system. This can be done by representing uncertain parameters as random vectors with a joint probability density function (PDF). This density function is known as the posterior distribution function. The use of Bayesian techniques becomes useful when an analytical solution to this function, the posterior, is not available. This is the case in FEM updating because of the high dimensional parameter search space.

Different methods have been proposed to estimate the PDF – the most common method being the maximum likelihood. This method represents an asymptotic approximation to the full Bayesian solution and it can be applied in the case where the unknown parameters are modeled as Gaussian. The most probable values are obtained by maximizing the likelihood function and the covariance matrix is obtained by using the Hessian of the likelihood function [9].

An alternative to the above is to use sampling techniques to estimate the PDF. Different methods such as Latin Hypercube Sampling (LHS) [14], Orthogonal Array Sampling [15] have been developed to predict complex distributions. The LHS method has been employed in model updating by Khodaparast [7]. Among sampling methods the most popular are the Markov chain Monte Carlo (MCMC) methods. These methods allow sampling from a large class of distributions and some of these algorithms cop well with the dimensionality of the sample space. The Metropolis–Hastings (MH) algorithm is the most common MCMC algorithm [16–19]. This method samples from a proposed PDF and the acceptance-rejection step is used to ensure that the Markov chain is reversible with respect to the stationary target density function [19]. The proposed PDF used to generate the samples forms a random walk trajectory during the search. In the MH algorithm smaller transitions can assure a high acceptance rate but a large amount of time is needed to cover more space during the search procedure. The random walk step becomes useless for complex systems where highly correlated samples can be obtained with a poor mixing of the chain and the MH acceptance rate will decrease exponentially. The MH algorithm has been implemented on the FEM updating problem [4,12,20].

Ching et al. [21] successfully applied another MC method called Gibbs sampling [22] to solve high-dimensional model updating problems. Based on the MH algorithm, Ching and Cheng [23] introduced the Transitional Markov Chain Monte Carlo (TMCMC) algorithm and Muto and Beck [24] applied it to the updating of hysteretic structural models. Cheung and Beck [11] applied the Hybrid Monte Carlo (HMC) method to update a linear structural dynamic model with 31 uncertain parameters. The probabilistic Bayesian model updating approach used in [11] was able to characterize modeling uncertainties associated with the underlying structural system. The HMC method promises the ability to solve higher-dimensional complex problems. The Monte Carlo trajectory it uses is guided by the derivative of the target log-density probability which leads towards areas of high probability during the searching process [25]. In this method the updated parameter vector is treated as a system displacement and an auxiliary variable, called the momentum vector, is introduced to construct a new Molecular Dynamic (MD) system. The total system energy – called the Hamiltonian function – is evaluated using the Störmer-Verlet (also called leapfrog integrator) algorithm. However, the Störmer–Verlet integrator does not conserve energy especially when the time step used and/or the system size are considered large. To overcome this limitation, a modified HMC algorithm called the Shadow Hybrid Monte Carlo (SHMC) was proposed in [26].

In this paper the SHMC is implemented for its proposed good-ability to sample the posterior PDF. This method is tested on updating FE models of an unsymmetrical H-shaped Structure and the GARTEUR SM-AG19 structure. The accuracy, efficiency and limitations of the SHMC technique are compared to those of the HMC algorithm on the same structures.

In the next section a short theoretical background of finite element model is presented. In Section 3 the posterior distribution function of the uncertain parameters of the FEM are presented. Section 4 introduces the HMC technique.