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Brief paper

Pseudo-predictor feedback stabilization of linear systems with time-varying input delays[☆]

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ABSTRACT

This paper is concerned with stabilization of (time-varying) linear systems with a single time-varying input delay by using the predictor based delay compensation approach. Differently from the traditional predictor feedback which uses the open-loop system dynamics to predict the future state and will result in an infinite dimensional controller, we propose in this paper a pseudo-predictor feedback (PPF) approach which uses the (artificial) closed-loop system dynamics to predict the future state and the resulting controller is finite dimensional and is thus easy to implement. Necessary and sufficient conditions guaranteeing the stability of the closed-loop system under the PPF are obtained in terms of the stability of a class of integral delay operators (systems). Moreover, it is shown that the PPF can compensate arbitrarily large yet bounded input delays provided the open-loop (time-varying linear) system is only polynomially unstable and the feedback gain is well designed. Comparison of the proposed PPF approach with the existing results is well explored. Numerical examples demonstrate the effectiveness of the proposed approaches.

1. Introduction

Since time delay can be a source of performance degradation and even instability of control systems (Hale, 1977), control of time-delay systems has attracted much attention for several decades and various problems that were initially solved for delay-free systems have been investigated in the time-delay setting (see, *e.g.*, Chen, Fu, Niculescu, & Guan, 2010, Cong & Zou, 2010, Fridman, 2002, Hale, 1977, Michiels & Niculescu, 2007, Richard, 2003, Wu, Shi, Su, & Chu, 2011, Wu, Shi, Su, & Chu, 2013 and the references cited therein). Stability analysis and stabilization of time-delay systems are two fundamental problems that are important in the other analysis and synthesis problems for time-delay systems. One of the most efficient method for handling asymptotic stability analysis and stabilization of time-delay systems is the Lyapunov–Krasovskii functional based method (see, *e.g.*, Chen & Zheng, 2011, Fridman, 2002, Lam, Xu, Ho, & Zou, 2012 and Xu, Lam, & Yang, 2001). The basic idea is to find a positive-definite functional such that its time-derivative along the trajectories of the time-delay system is negative-definite. The results obtained by this kind of methods for stability analysis can be easily recast into linear matrix inequalities and can also be easily adopted to the stabilizing controllers design. However, only sufficient conditions can be obtained by this approach. On the other hand, control of time-varying linear systems has been recognized as a very important and difficult problem (Anderson, Ilchmann, & Wirth, 2013; Zhao & Chen, 2009; Zhao & Zhou, 2012) and only a few results are available in the literature concerned with time-varying linear systems with delays (Zhou, 2014).

Predictor feedback is another efficient approach to dealing with the control of time-delay systems, particularly, input delayed systems (Artstein, 1982; Kojima, Uchida, Shimemura, & Ishijima, 1994; Krstic, 2009; Manitius & Olbrot, 1979; Mondie & Michiels, 2003; Richard, 2003). Compared with the Lyapunov–Krasovskii functional based method which may be *passive* in most cases, the predictor feedback approach is *active* in the sense that effort is made to compensate the delay effect completely or partially. This approach is even effective when the delays are time-varying and the plant dynamics are nonlinear. In recent years, predictor

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feedback approaches for controlling time-delay systems have received renewed interest and a couple of new results has been reported in the literature. For example, in Krstic (2009) the PDE backstepping approach is utilized to design the boundary control laws that is equivalent to the classical predictor feedback controller by modeling the input delay dynamics as a partial differential equation (PDE) of transport type, and in Zhou (2014) and Zhou, Lin, and Duan (2012) a truncated predictor feedback approach is originally established to avoid the implementation difficulty of the traditional predictor feedback controllers for input delayed systems by safely dropping out the distributed terms if the open-loop system is only polynomially unstable. For more related work on predictor-type controllers for time-delay systems and their implementation issues, see Gu (2012), Krstic (2009), Zhong (2004), Zhou et al. (2012) and the references cited there.

In this paper, we propose a new predictor feedback approach for stabilization of (time-varying) linear systems with a single (large) time-varying input delay. Differently from the traditional predictor feedback which uses the open-loop system dynamics to predict the future state and will result in an infinite dimensional controller, the established predictor approach uses the *artificial closed-loop* system dynamics to predict the future state and the resulting controller is finite dimensional and is thus easy to implement. Since the used closed-loop system is *artificial*, the proposed method is referred to as pseudo-predictor feedback (PPF) approach. We also provide necessary and sufficient conditions guaranteeing the stability of the closed-loop system under the PPF in terms of the stability of a class of integral delay operators (systems). A Lyapunov–Krasovskii functional is also constructed if a Lyapunov–Krasovskii functional for the integral delay operator (system) is available. Based on a sufficient condition for the stability of the integral delay operator (system), we show that the PPF can compensate arbitrarily large yet bounded input delays provided the open-loop (periodic time-varying) system is only polynomially unstable and the feedback gain is well designed. Explicit sufficient condition is also provided to guarantee the exponential stability if the system is time-invariant. Finally, the advantages of proposed PPF approach over the existing ones are well explored. Numerical examples demonstrate the effectiveness of the proposed approaches.

The remainder of this paper is organized as follows. The problem formulation and some preliminaries are shown in Section 2. The PPF approach in the general setting is then developed in Section 3. The particular case that the open-loop system is only polynomially unstable is investigated detailedly in Section 4. Numerical examples are worked out in Section 5 to validate the effectiveness of the proposed approaches and Section 6 concludes the paper. Finally, a class of parametric differential Riccati equations used in Section 4 are studied in detail in the Appendix.

2. Problem formulation and preliminaries

Consider a time-varying linear system with time-varying input delay

$$\dot{x}(t) = A(t)x(t) + B(t)u(\phi(t)), \quad t \geq t_0, \tag{1}$$

where $A(t) : [t_0, \infty) \to \mathbf{R}^{n\times n}$ and $B(t) : [t_0, \infty) \to \mathbf{R}^{n\times m}$ are, respectively, the system and control matrices and are such that, for all $t \geq t_0$,

$$\|A(t)\| \leq a < \infty, \qquad \|B(t)\| \leq b < \infty, \tag{2}$$

and $\phi(t) : [t_0, \infty) \to \mathbf{R}$ is a continuously differentiable function that incorporates the actuator delay. The function $\phi(t)$ can be defined in a more standard form

$$\phi(t) = t - \tau(t), \tag{3}$$

where $\tau(t) : [t_0, \infty) \to [0, \infty)$ is the time-varying delay.

Assumption 1. The function $\phi : [t_0, \infty) \to \mathbf{R}$, is continuously differentiable, invertible and exactly known and is such that

$$0 < \phi_- \leq \dot{\phi}(t) < \infty, \quad \forall t \in [t_0, \infty], \tag{4}$$

and the delay $\tau(t)$ is bounded, namely, there exists a finite number $T \geq 0$ such that

$$0 \leq \tau(t) \leq T, \quad \forall t \in [t_0, \infty]. \tag{5}$$

Condition (4) ensures that the inverse function of $\phi(t)$, denoted by $\phi^{-1}(t)$, exists for all $t \geq t_0$, which is necessary for the construction of predictor feedback control for system (1) (see, for example, Krstic, 2010 and Zhou et al., 2012).

We next introduce the traditional predictor feedback control of system (1). Assume that there exists a (time-varying) feedback gain $F(t) : [t_0, \infty) \to \mathbf{R}^{m\times n}$ such that $\dot{\upsilon}(t) = (A(t) + B(t)F(t))\upsilon(t)$ is exponentially stable (discussions on the existence and design of $F(t)$ can be found in the textbook Rugh, 1996 and the recent paper Anderson et al., 2013). If we design the artificial control

$$u(t) = F\left(\phi^{-1}(t)\right)x\left(\phi^{-1}(t)\right), \quad t \geq \phi^{-1}(t_0), \tag{6}$$

the closed-loop system consisting of (1) and (6) will take the form

$$\dot{x}(t) = (A(t) + B(t)F(t))x(t), \quad t \geq \phi^{-1}(t_0), \tag{7}$$

and is thus exponentially stable. As (6) is acausal, to make it to be implementable, we may predict $x\left(\phi^{-1}(t)\right)$ from $x(t)$ by using the open-loop system (1) as

$$x\left(\phi^{-1}(t)\right) = \Phi_A\left(\phi^{-1}(t), t\right)x(t) + \varphi(t), \tag{8}$$

where $\Phi_A(t, s)$ is the state transition matrix for system (1) with $u(t) \equiv 0$ and $\varphi(t) = \int_t^{\phi^{-1}(t)} \Phi_A(\phi^{-1}(t), s)B(s)\, u(\phi(s))\mathrm{d}s$. Substituting (8) into (6) gives the following predictor feedback

$$u(t) = F\left(\phi^{-1}(t)\right)\left(\Phi_A\left(\phi^{-1}(t), t\right)x(t) + \varphi(t)\right). \tag{9}$$

The predictor feedback controller (9) is infinite dimensional as it involves the integration of $u(t)$ in the interval $[\phi(t), t]$, which makes the implementation hard even when the delay $\tau(t)$ and the system matrices $A(t)$ and $B(t)$ are constant (Richard, 2003; Van Assche, Dambrine, Lafay, & Richard, 1999). Recently, we have proposed a truncated version of the predictor feedback (9) by neglecting the distributed term $\varphi(t)$ in it to yield

$$u_{\mathrm{TPF}}(t) = F\left(\phi^{-1}(t)\right)\Phi_A\left(\phi^{-1}(t), t\right)x(t), \tag{10}$$

which is referred to as truncated predictor feedback (TPF).

Lemma 1 (*Zhou, 2014 and Zhou et al., 2012*). *Assume that A and B are constant, namely, system (1) becomes*

$$\dot{x}(t) = Ax(t) + Bu(\phi(t)), \quad t \geq t_0, \tag{11}$$

where $\phi(t)$ satisfies Assumption 1. Assume also that (A, B) is controllable and all the eigenvalues of A are on the imaginary axis. Let $F = -B^{\mathsf{T}}P$ where P is the unique positive definite solution to the following algebraic Riccati equation (ARE)

$$A^{\mathsf{T}}P + PA - PBB^{\mathsf{T}}P = -\gamma P, \tag{12}$$

where $\gamma > 0$ is a parameter. Then, for any $T > 0$, system (11) is stabilized by the TPF (10), namely,

$$u_{\mathrm{TPF}}(t) = Fe^{A(\phi^{-1}(t)-t)}x(t) = -B^{\mathsf{T}}Pe^{A(\phi^{-1}(t)-t)}x(t), \tag{13}$$

where $\gamma \in (0, \frac{\delta^*}{T(n-1)})$ with δ^* being the unique positive root of $\frac{(n-1)^2}{n^3} = \delta e^{\delta}\left(e^{\delta} - 1\right)$ and $n \geq 2$.