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Brief paper

Saturation-based switching anti-windup design for linear systems with nested input saturation[☆]

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ABSTRACT

This paper proposes a saturation-based switching anti-windup design for the enlargement of the domain of attraction of a linear system subject to nested saturation. A nestedly saturated linear feedback is expressed as a linear combination of a set of auxiliary linear feedbacks, which form a convex hull where the nestedly saturated linear feedback resides. This set of auxiliary linear feedbacks is then partitioned into several subsets. The auxiliary linear feedbacks in each of these subsets form a convex sub-hull of the original convex hull. When the value of the nestedly saturated linear feedback falls into a convex sub-hull, it can be expressed as a linear combination of the subset of all the auxiliary feedbacks that form the convex sub-hull. A separate anti-windup gain is designed for each convex sub-hull by using a common quadratic Lyapunov function and is implemented when the value of the nestedly saturated linear feedback falls into this convex sub-hull. Simulation results indicate that such a saturation-based switching anti-windup design has the ability to significantly enlarge the domain of attraction of the closed-loop system.

1. Introduction

Dynamical systems subject to nested saturation in their input have received significant interest from control system researchers due to their frequent occurrence in various engineering applications. As an example, control systems subject to simultaneous actuator magnitude and rate saturation in the input (Bateman & Lin, 2003; Berg, Hammett, Schwartz, & Banda, 1996; Nguyen & Jabbari, 2000; Tyan & Bernstein, 1997) can be modeled with nested saturation. Works on the topic of stability and stabilization of linear systems subject to nested saturation in the input have emerged in a large number in the past two decades. In another example, nonlinear feedback laws of nested saturation type have been utilized to achieve global asymptotic stabilization for linear systems subject to actuator saturation (Sussmann, Sontag, & Yang, 1994; Teel, 1992), while linear feedback has been proven unable to do this.

If not taken into account in the design of the system, nested saturation in general will degrade the performance of the system, and may even cause the system to lose its stability. As a result, properly handling nested saturation has been an important issue in stability analysis and stabilization of linear systems subject to nested saturation in the input. A popular approach to dealing with nested saturation is to treat it as a sector nonlinearity (Tarbouriech, Prieur, & Gomes da Silva, 2006; Wu & Soto, 2003). Based also on the sector-based analysis of nested saturation, stability and the domain of attraction have been studied (Tarbouriech et al., 2006). Another effective method for handling nested saturation is the linear differential inclusion approach (Bateman & Lin, 2003; Fiacchini, Tarbouriech, & Prieur, 2012; Zhou, Zheng, & Duan, 2011), which puts a nestedly saturated linear feedback inside a convex hull of a set of auxiliary linear feedbacks. The linear differential inclusion approach, in general, is less conservative than the sector-based approach.

Anti-windup is an important approach to mitigating the adverse effect of actuator saturation. Its objective is to recover as much as possible the performance of the closed-loop system in the absence of actuator saturation (Grimm et al., 2003; Wu & Soto, 2003; Zaccarian & Teel, 2004). An important performance to consider in the anti-windup design is the size of the domain of attraction of the resulting closed-loop system (Cao, Lin, & Ward, 2002;

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Gomes da Silva & Tarbouriech, 2005; Li & Lin, 2013b; Lv & Lin, 2010). Gomes da Silva and Tarbouriech (2005) took the sector-based approach and formulated the problem of designing an anti-windup gain for the enlargement of the domain of attraction into an optimization problem with linear matrix inequality (LMI) constraints. The convex hull representation of saturated linear feedbacks was adopted by Cao et al. (2002), Li and Lin (2013b) and Lv and Lin (2010) and iterative linear matrix inequality algorithms were developed to compute the anti-windup gains. In particular, a saturation-based switching anti-windup scheme was proposed by Li and Lin (2013b) to obtain a significantly larger domain of attraction than those methods of Cao et al. (2002), Gomes da Silva and Tarbouriech (2005) and Lv and Lin (2010). This anti-windup design partitions the convex hull that represents the saturated linear feedback into several convex sub-hulls, designs a separate anti-windup gain for each convex sub-hull and switches among the different anti-windup gains according to which convex sub-hull the value of the saturated linear feedback falls into.

Following the idea of designing the saturation-based switching anti-windup compensator to enlarge the domain of attraction of a linear system subject to actuator saturation (Li & Lin, 2013b), in this paper, we consider the same problem of designing saturation-based switching anti-windup gains when the system is subject to nested saturation, instead of single layer saturation, in the input. The first contribution of this paper is to propose an improved convex hull representation for a nestedly saturated linear feedback, which has the same form as the existing representation (Fiacchini et al., 2012; Zhou et al., 2011), but contains more decision variables. As a result, less conservative conditions can be established to characterize the invariance of an ellipsoid as the estimate of the domain of attraction. Second, we present a partitioning of the set of auxiliary linear feedbacks that define the improved convex hull into several subsets. The auxiliary linear feedbacks in each of these subsets form a convex sub-hull of the improved convex hull. When the value of the nestedly saturated linear feedback falls into a convex sub-hull, it can be expressed as a linear combination of the subset of all the auxiliary linear feedbacks that define the convex sub-hull. A separate anti-windup gain is designed for each convex sub-hull by using a common quadratic Lyapunov function and is implemented when the value of the nestedly saturated linear feedback falls into this convex sub-hull. Numerical simulation will be carried out to demonstrate that such a saturation-based switching anti-windup design has the ability to result in a significantly larger domain of attraction than the single anti-windup design.

The remaining part of our paper is organized as follows. In Section 2, the problem to be studied in the paper is stated and the improved convex hull representation of the nestedly saturated linear feedback is proposed that reduces the conservativeness of the existing treatments of nested saturation (Bateman & Lin, 2003; Fiacchini et al., 2012; Zhou et al., 2011). Based on this new convex hull representation of the nestedly saturated linear feedback law, we will present the proposed saturation-based switching anti-windup design in Section 3. Numerical examples are presented in Section 4 to illustrate that significantly larger estimates of domain of attraction can be obtained by using the proposed saturation-based switching anti-windup compensator. Section 5 concludes the paper.

We will use standard notation. Let $\mathrm{sat}: \mathbf{R}^m \to \mathbf{R}^m$ denote the vector valued standard saturation function, which is defined as $\mathrm{sat}(u) = [\mathrm{sat}(u_1), \mathrm{sat}(u_2), \dots, \mathrm{sat}(u_m)]^{\mathrm{T}}$, $\mathrm{sat}(u_i) = \mathrm{sgn}(u_i)\min\{1, |u_i|\}$. For an $F \in \mathbf{R}^{m\times n}$, let $\mathcal{L}(F) = \{x \in \mathbf{R}^n : |f_i x|_\infty \le 1, i \in I[1, m]\}$, where f_i represents the ith row of matrix F. We note that $\mathcal{L}(F)$ represents the region in $\mathbf{R}^n$ where Fx does not saturate. Also, for an integer m_k, let $\mathcal{D}_k$ be the set of $m_k \times m_k$ diagonal matrices whose diagonal elements are either 1 or 0. There are 2^{m_k} elements in $\mathcal{D}_k$. Suppose that these elements of $\mathcal{D}_k$ are labeled as D_{i_k}, $i_k \in I[1, 2^{m_k}]$. Here and throughout this paper, for two integers l_1 and l_2, $I[l_1, l_2]$ denotes the set of integers $\{l_1, l_1 + 1, \dots, l_2\}$. Denote that $D_{i_k}^- = I - D_{i_k}$. Clearly, $D_{i_k}^- \in \mathcal{D}_k$ if $D_{i_k} \in \mathcal{D}_k$. For a positive definite $P \in \mathbf{R}^{n\times n}$, $\mathcal{E}(P) := \{x \in \mathbf{R}^n : x^{\mathrm{T}}Px \le 1\}$. For a square matrix A, $\mathrm{He}(A) := A^{\mathrm{T}} + A$.

2. A new convex hull representation of nested saturation and the anti-windup design problem

2.1. Convex hull representation of a nestedly saturated linear feedback

We first recall from Li and Lin (2013a) an alternative convex hull representation of a saturated linear feedback law, which generalizes a representation in Hu and Lin (2001).

Lemma 1. *Let $F, H_i \in \mathbf{R}^{m\times n}$, $i \in I[1, 2^m]$. For an $x \in \mathbf{R}^n$, if $x \in \mathcal{L}(H_i)$, $i \in I[1, 2^m]$, then*

$$\mathrm{sat}(Fx) \in \mathrm{co}\{D_iFx + D_i^- H_i x : i \in I[1, 2^m]\}, \tag{1}$$

where co *stands for the convex hull.*

Clearly, Lemma 1 will reduce to the related result in Hu and Lin (2001) if we set $H_i = H$ for all $i \in I[1, 2^m]$. An equivalent treatment of saturated linear feedbacks, not in the form of convex hull representation, was earlier proposed by Alamo, Cepeda, and Limon (2005), and other equivalent results can be found in Fiacchini et al. (2012), Zhou (2013) and Zhou et al. (2011).

In this paper, we are concerned with the following linear system subject to nested saturation in the input:

$$\begin{aligned}\dot{x} = {} & Ax + B_1\mathrm{sat}(F_1x + B_2\mathrm{sat}(F_2x \\ & + B_3\mathrm{sat}(F_3x + \cdots + B_q\mathrm{sat}(F_qx)))),\end{aligned} \tag{2}$$

where $A \in \mathbf{R}^{n\times n}$, $B_k \in \mathbf{R}^{m_{k-1}\times m_k}$ and $F_k \in \mathbf{R}^{m_k\times n}$, $k \in I[1, q]$, $m_0 = n$. Systems of this form were originally considered by Bateman and Lin (2003). Throughout the paper, we will number the saturation functions from the outmost layer inward, with the outmost layer as the first layer saturation function. Define a set of auxiliary matrices $H_k(i_1, i_2, \dots, i_q) \in \mathbf{R}^{m_k\times n}$, $(i_1, i_2, \dots, i_q) \in \Pi$, where $\Pi = (I[1, 2^{m_1}] \times [1, 2^{m_2}] \times \cdots \times I[1, 2^{m_q}])$, for the kth layer saturation function. For a fixed k, there are $2^{\sum_{r=1}^{q} m_r}$ such auxiliary matrices. Following the approach to expressing the saturated linear feedback on the convex hull of a group of auxiliary linear feedback laws, as described in Lemma 1, we can establish the following lemma that provides a similar treatment to nested saturation found in (2).

Lemma 2. *For an $x \in \mathbf{R}^n$, if $x \in \mathcal{L}(H_k(i_1, i_2, \dots, i_q))$, $k \in I[1, q]$, $(i_1, i_2, \dots, i_q) \in \Pi$,*

$$\begin{aligned}&\mathrm{sat}(F_1x + B_2\mathrm{sat}(F_2x + B_3\mathrm{sat}(F_3x + \cdots + B_q\mathrm{sat}(F_qx)))) \\ &\quad \in \mathrm{co}\Bigg\{\sum_{k=1}^{q}\left(\prod_{l=1}^{k-1} D_{i_l}B_{l+1}\right) D_{i_k}F_kx + \sum_{k=1}^{q}\left(\prod_{l=1}^{k-1} D_{i_l}B_{l+1}\right) \\ &\qquad \times D_{i_k}^- H_k(i_1, i_2, \dots, i_q)x : (i_1, i_2, \dots, i_q) \in \Pi\Bigg\},\end{aligned} \tag{3}$$

where we have defined $\prod_{l=1}^{0} D_{i_l}B_{l+1} = I$.

Proof. Let

$$\begin{aligned} v_1 &= F_1x + B_2\mathrm{sat}(F_2x + B_3\mathrm{sat}(F_3x + \cdots + B_q\mathrm{sat}(F_qx))), \\ v_2 &= F_2x + B_3\mathrm{sat}(F_3x + \cdots + B_q\mathrm{sat}(F_qx)), \\ &\ \vdots \\ v_q &= F_qx. \end{aligned}$$