Mechanical Systems and Signal Processing 52-53 (2015) 495–505

Contents lists available at ScienceDirect

Mechanical Systems and Signal Processing

journal homepage: www.elsevier.com/locate/ymssp

# Forced response approach of a parametric vibration with a trigonometric series

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ARTICLE INFO

*Article history:*
Received 14 December 2012
Received in revised form
20 March 2014
Accepted 27 June 2014
Available online 17 July 2014

*Keywords:*
Parametric vibration
Forced response
Trigonometric series

ABSTRACT

A forced vibration problem with parametric stiffness is modeled by feedback structure in this manuscript, and the forced response is expressed as a special trigonometric series. The forced response of this problem is determined by algebraic equation. By applying harmonic balance and limitation operation, all coefficients of the harmonic components in the forced response solution are fully approached. The investigation result shows that the new approach has an advantage in the computational time and accuracy, and it is very significant for the theoretical research and engineering application in dealing with the problem of forced parametric vibration.



## 1. Introduction

The problem of parametric vibration arises in many branches of physics and engineering, and the investigation of stability and response prediction are the two most significant dynamic problems in the parametric vibration system. In the past, several methods have been used to study the stability of systems with periodic coefficients. These include Hill's method [1], the perturbation method [2] and the averaging approach, the Floquet theory with numerical integration [3], Sinha's numerical scheme with the shifted Chebyshev polynomial [4,5], etc.

Many computation methods, such as the J.W. David's transfer matrix method [6], linear combination of the Floquet eigenvectors [7], the improved direct spectral method [8], the multiple scales method [9–11], etc., are studied to find the forced responses. These computation formulations can be used to efficiently find the forced responses in a multiple degree of freedom system with periodic coefficients. However, in science and engineering practice, it will be convenient to express the forced responses in the form of Fourier series, which is very useful in an online vibration monitoring system. For example, in the mechanical fault diagnosis for a rotor with a cross crack, such an expression is important and not replaceable [12]. However, so far no related response solution has directly been considered in all of the above approaches.

In this paper, the method of modulation feedback [13] is extended to investigate a forced response whose solution form is similar to Fourier series. Using harmonic balance in the equation and limitation operation, the full expression of steady state forced response is conducted. Although the scope comes from the content of a modulation system, it reflects the intrinsic physical character of frequency splitting in the parametric vibration system.

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http://dx.doi.org/10.1016/j.ymssp.2014.06.013
0888-3270/

## 2. Modulation feedback conception

Consider a system excited by both periodic coefficients and external force terms that have different periods as described in the following equation:

$$\frac{d^2x}{dt^2}+\omega_n^2(1+\beta\ \cos\ \omega_o t)x=A\ \cos\ \omega_p t, \tag{1}$$

The above equation can be written as

$$\frac{d^2x}{dt^2}+\omega_n^2 x=A\ \cos\ \omega_p t-x\omega_n^2\beta\ \cos\ \omega_o t, \tag{2}$$

From Eq. (2), the problem of a forced parametric vibration can be schematically described in Fig. 1, a special feedback system that contains a second-order linear system and an amplitude modulation. The output of the system is a response of the forced parametric vibration.

Because of the amplitude modulation, the phenomenon of frequency splitting occurs in the system, and it can be stated as: component $\omega_p$ splits into the combination of harmonic components $\omega_p-\omega_o$ and $\omega_p+\omega_o$ by a modulation operation within the first $\Delta t$. The modulation result feeds back directly to the second-order linear system as an input of the new harmonic components. Thus, within the second $\Delta t$, three components $\omega_p$, $\omega_p-\omega_o$ and $\omega_p+\omega_o$ will be the output of the system and be involved in the next splitting operation, a sequential combination of five harmonic components $\omega_p$, $\omega_p-\omega_o$, $\omega_p-2\omega_o$, $\omega_p+\omega_o$ and $\omega_p+2\omega_o$ will become the modulation results and will be fed back to the second-order linear system, and the iterate splitting operation continues. The whole physical procedure of the frequency splitting is schematically described in Fig. 2.

Assuming the process continues, the frequency splitting will eventually reach a state of balance. Therefore, as a result of the frequency splitting, there are several linear combinations of the harmonic components ($\omega_p$ and $\omega_o$) in the system output, and they can be mathematically expressed as follows:

$$x(t)=\sum_{k=-\infty}^{\infty} B_k \cos(\omega_p+k\omega_o)t. \tag{3}$$

Because the energy of the harmonic components is limited, and it concentrated on the narrow band range around the frequency $\omega_p$, the coefficient $B_k \to 0$ while $k \to \infty$.

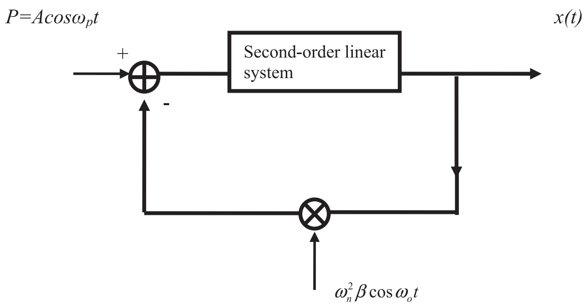


**Fig. 1.** Modulation feedback system.

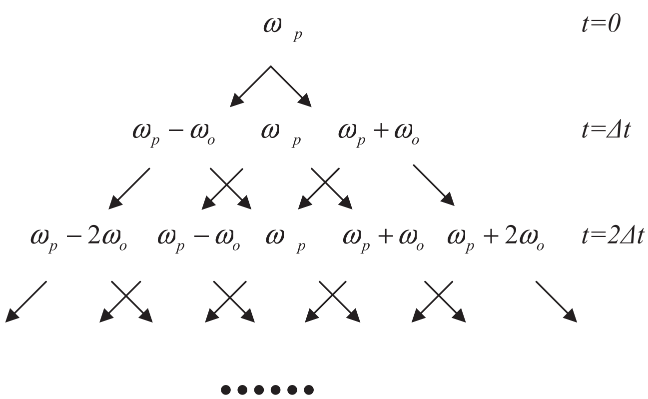


**Fig. 2.** Frequency splitting procedure ($t \geq 0$, $\Delta t \to 0$).