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Brief paper

Adaptive observers for a class of uniformly observable systems with nonlinear parametrization and sampled outputs[☆]

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ABSTRACT

In this paper, we propose an adaptive observer for a class of uniformly observable nonlinear systems with nonlinear parametrization and sampled outputs. A high gain adaptive observer is first designed under the assumption that the output is continuously measured and its exponential convergence is investigated, thanks to a well defined persistent excitation condition. Then, we address the case where the output is available only at (non uniformly spaced) sampling instants. To this end, the continuous-time output observer is redesigned leading to an impulsive observer with a corrective term involving instantaneous state impulses corresponding to the measured samples and their estimates. Moreover, it is shown that the proposed impulsive observer can be put under the form of a hybrid system composed of a continuous-time observer coupled with an inter-sample output predictor. Two design features are worth to be emphasized. Firstly, the observer calibration is achieved through the tuning of a scalar design parameter. Secondly, the exponential convergence to zero of the observation and parameter estimation errors is established under a well defined condition on the maximum value of the sampling partition diameter. More specifically, the observer design is firstly carried out in the case of linear parametrization before being extended to the nonlinear one. The theoretical results are corroborated through simulation results involving a typical bioreactor.

1. Introduction

Over the last decades, design of adaptive observers has become a wide and active research field as pointed out by its underlying literature. The adaptive observers perform a simultaneous estimation of the state variables and the unknown parameters. These observers are particularly used in challenging applications, namely the adaptive control and fault detection and isolation. The seminal contributions related to adaptive observer designs have been devoted to linear time invariant systems (see Kreisselmeier, 1977 and references therein). The case of linear time-varying systems has been investigated in recent works within deterministic and stochastic frameworks (Perabò & Zhang, 2009; Zhang, 2002). Several approaches have been adopted to tackle the design of adaptive observers for nonlinear systems. The usual approach is based on appropriate coordinate transformations which allow to obtain linear error dynamics up to an output injection (Bastin & Gevers, 1988; Santosuosso, Marino, & Tomei, 2001). An optimization based approach has been presented in Cho and Rajamani (1997) assuming the feasibility of a set of linear matrix inequalities. Some results have been proposed in Besançon (2000) and Stamnes, Aamo, and Kaasa (2011) without requiring the considered nonlinear systems to be linearizable. These results have been established assuming the existence of some Lyapunov functions satisfying particular conditions. Adaptive versions of high gain observers have been proposed in De Leon Morales, Besançon, and Huerta-Guevara (2006) and Farza, M'Saad, Maatoug, and Kamoun (2009). Though most contributions on design of adaptive observers deal with linear parametrization, there are nevertheless results dealing with nonlinear parameterizations (see Farza et al., 2009; Grip, Saberi, & Johansen, 2011; Kojić & Annaswamy, 2002; Tyukin, Steur, Nijmeijer, & van Leeuwen, 2013 and references therein). In all the

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above cited contributions, the output measurements are assumed to be continuously available when designing the observer. This is more an exception than a rule in the real world life where the measurements are generally available only at the sampling instants. Such an issue has been particularly addressed for the observer design without parameter adaptation. To this end, the continuous-time output observers are redesigned to derive sampled data observers which are also referred to as continuous-discrete time observers. These observers provide the state estimates from output measurements that are available only at sampling instants. Several approaches have been pursued when designing these observers, namely a high gain approach (Andrieu & Nadri, 2010; Deza, Busvelle, Gauthier, & Rakotopara, 1992; Nadri, Hammouri, & Grajales, 2013), an output predictor approach (Karafyllis & Kravaris, 2009), and an LMI-based approach (Raff, Kögel, & Allgöwer, 2008).

In this paper, we propose a sampled output high gain adaptive observer for a class of uniformly observable single-input–single-output nonlinear systems. This observer is presented under the form of an impulsive system issued from the redesign of a high gain adaptive observer working with a continuous-time output. The latter is similar to the one proposed in Farza et al. (2009) up to an adequate modification of the persistent excitation requirement. Indeed, the new persistent excitation condition is admissible unlike the one considered in Farza et al. (2009). After presenting the sampled output adaptive observer under an impulsive form, we show that it can be put under the form of a hybrid system constituted by a continuous-time output like observer where the sampled output is replaced by an appropriate prediction provided by a dynamical system that has to be solved between two successive sampling instants.

There are three specific features of the proposed sampled output adaptive observer that should be emphasized with respect to the available ones. The first one lies in the fact that it constitutes a seminal contribution to the adaptive observer within a sampled output framework design since the output is assumed to be measured in a continuous way in most of the available adaptive observers. The second feature concerns the simplicity of the observer from both the observer gain updating as well as the implementation points of view. Such a simplicity is intrinsic to the structure of the underlying continuous-time output observer. The third feature consists in an adequate convergence analysis approach which is simple and fruitful from both fundamental and practical points of view. Indeed, it provides precise expressions of the upper bound of the sampling partition diameter as well as the rate of exponential convergence of the observation and parameter estimation error.

The paper is organized as follows. Section 2 is devoted to the problem statement with a particular emphasis on the considered class of systems and the classical high gain assumptions. In Section 3, the design of the underlying continuous-time output high gain adaptive observer is comprehensively presented with a particular emphasis on the main steps of its convergence analysis. The main contribution of the paper is given in Section 4. The sampled output adaptive observer is first introduced under an impulsive form and its exponential convergence analysis is carried out thanks to the main outlines of the proof given in the continuous-time output case together with an appropriate technical lemma. Then, it is shown that the proposed adaptive observer can be rewritten under a useful hybrid form, revealing thereby the close relationship with its underlying continuous-time output high gain adaptive observer. In Section 5, the proposed adaptive observer design is revisited in the nonlinear parametrization case up to appropriate assumptions. The effectiveness of the proposed adaptive observer is corroborated by promising simulation results involving a typical bioreactor in Section 6. Finally, some concluding remarks are given in Section 7.

Throughout the paper, $\mathbb{R}^{+\star}$ denotes the set of positive scalars, I_p and 0_p denote the p-dimensional identity and zero matrices respectively, $\|\cdot\|$ denotes the euclidian norm and $\lambda_M(\cdot)$ (resp. $\lambda_m(\cdot)$) will be used to denote the largest (resp. the smallest) eigenvalue of $(\cdot)$.

2. Problem statement

The ultimate motivation of this paper consists in providing a suitable framework to design a sampled output high gain adaptive observer for single-input–single-output systems which are diffeomorphic to the following form:

$$\begin{cases} \dot{x}(t) = Ax(t) + g(u(t), x(t)) + \Psi(u(t), x(t))\rho \\ y(t_k) = Cx(t_k) = x_1(t_k) \end{cases} \tag{1}$$

$$A = \begin{pmatrix} 0 & I_{n-1} \\ 0 & 0 \end{pmatrix}, \qquad C = \begin{pmatrix} 1 & 0 & \dots & 0 \end{pmatrix} \tag{2}$$

where $x(t) = (x_1 \ \dots \ x_n)^T \in \mathbb{R}^n$ and $\rho = (\rho_1 \ \dots \ \rho_m)^T \in \mathbb{R}^m$ denote the state and the unknown parameters of the system respectively, $u(t) \in \mathbb{R}$ is the input of the system, $y \in \mathbb{R}$ denotes the system output that is available only at the sampling times t_k that satisfy $0 \leq t_o < \cdots < t_k < t_{k+1} < \cdots$ with time-varying intervals $\tau_k = t_{k+1} - t_k$ and $\lim_{k\to\infty} t_k = +\infty$, g is a nonlinear vector function that has a triangular structure with respect to x, i.e. $g(u, x) = (g_1(u, x_1)\ g_2(u, x_1, x_2)\ \dots\ g_n(u, x))^T$, Ψ is a nonlinear matrix function of dimension $n \times m$ each column of which is a vector function with a triangular structure with respect to x.

In the case where the parameter vector ρ is known, it has been shown in Gauthier, Hammouri, and Othman (1992) that the canonical form (1) characterizes the class of single-input–single-output systems that are observable for any input and for which a high gain observer has been proposed (see for instance Gauthier et al., 1992). When the parameter vector ρ is unknown, system (1) belongs to the class of systems considered in Farza et al. (2009) for which the authors proposed a high gain adaptive observer for the estimation of the state and the unknown parameters. The related exponential convergence to zero of the state and parameter estimation errors is guaranteed provided that a specific persistent excitation condition is satisfied. In this paper, the adaptive observer design proposed in Farza et al. (2009) is revisited under an appropriate persistent excitation requirement. The design is first carried out assuming that the output measurements are available in a continuous way as in Farza et al. (2009). Then, the proposed observer design is reconsidered under the assumption that the output is available only at (non uniformly) sampling instants, leading thereby to the main contribution of this paper. The resulting adaptive observer is an impulsive system constituted by two additive terms. The first one is a copy of the system, while the second one is a time varying corrective term depending on the last output samples.

The observer design requires some assumptions that will be stated later. Now standard high gain observer design assumptions are provided.

- A1 The state $x(t)$, the control $u(t)$ and the unknown parameters ρ are bounded, i.e. $x(t) \in X$, $u(t) \in U$ and $\rho \in \Omega$ where $X \subset \mathbb{R}^n$, $U \subset \mathbb{R}$ and $\Omega \in \mathbb{R}^m$ are compacts sets.
- A2 The matrix $\Psi(u, x)$ is continuous on $U \times X$.
- A3 The functions g and Ψ are Lipschitz with respect to x uniformly in u where $(u, x) \in U \times X$. Their Lipschitz constants will be denoted by L_g and L_Ψ, respectively.

Since the state is confined to the bounded set X, one can assume that Lipschitz prolongations of the nonlinearities, using smooth saturation functions, have been carried out and that the functions g and Ψ are provided from these prolongations. This allows to