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Design of sensor networks for instantaneous inversion of modally reduced order models in structural dynamics

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ABSTRACT

In structural dynamics, the forces acting on a structure are often not well known. System inversion techniques may be used to estimate these forces from the measured response of the structure. This paper first derives conditions for the invertibility of linear system models that apply to any instantaneous input estimation or joint input-state estimation algorithm. The conditions ensure the identifiability of the dynamic forces and system states, their stability and uniqueness. The present paper considers the specific case of modally reduced order models, which are generally obtained from a physical, finite element model, or from experimental data. It is shown how in this case the conditions can be directly expressed in terms of the modal properties of the structure. A distinction is made between input estimation and joint input-state estimation. Each of the conditions is illustrated by a conceptual example. The practical implementation is discussed for a case study where a sensor network for a footbridge is designed.

1. Introduction

The dynamic forces acting on a structure and the corresponding system states are of great importance to many engineering applications. Often, however, the dynamic forces and resulting system states can hardly be obtained by direct measurements, e.g. for wind loads, and have to be determined indirectly from dynamic measurements of the system response using system inversion techniques.

Originally, force identification and state estimation problems were treated separately. Force identification problems were initially solved off-line in a deterministic setting. Many methods were proposed, most of them based on the inversion of the frequency response function [1–3] or making use of a time domain approach [4–7]. Several state estimation algorithms have been proposed for linear as well as for non-linear systems [8–11]. A recursive deterministic method was presented by Klinkov and Fritzen [12], estimating both the input and system states from a set of output measurements. Currently, the attention is shifted to the development of recursive combined deterministic-stochastic approaches [13,14]. These methods do not only account for measurement errors, but also for modeling errors and additional unknown vibration sources. An algorithm was proposed by Gillijns and De Moor, where the input estimation is performed prior to the state estimation step [15].

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The algorithm was introduced in structural dynamics by Lourens et al. [16], extending the algorithm for use with reduced-order models. A similar approach was proposed by Niu et al. [17]. Alternatively, the dynamic forces and system states can be jointly estimated using a classical Kalman filter, hereby including the unknown forces in an augmented state vector [18].

This paper focuses on instantaneous system inversion, i.e. inversion without any time delay, covering the majority of inversion algorithms applied in structural dynamics. The invertibility of a system in general depends on three conditions. Firstly, the dynamic forces and/or the corresponding states must be identifiable from the given set of response measurements. Secondly, the system inversion algorithm must be stable, such that small perturbations of the data do not give rise to unbounded errors on the identified quantities. Thirdly, the estimates obtained must be uniquely defined by the measurement data. In the literature, the main requirements on the system description for instantaneous invertibility are extensively documented for the general case of linear systems [19–21]. For the specific case of linear modally reduced order models, which are often used in structural dynamics, the general conditions can be directly translated into a number of requirements on the sensor network, i.e. sensor types, sensor locations, and number of sensors.

The outline of this paper is as follows. In Section 2, the problem of system inversion is outlined. Next, in Sections 3–5, the requirements on the sensor network are derived, starting from the general conditions for system invertibility, as given in the literature. Section 6 discusses the practical implementation of the requirements for a case study, where a sensor network for a footbridge is designed that allows for the identification of multiple forces on the bridge deck. Finally, in Section 7, the work is summarized.

2. Problem formulation

In structural dynamics, first principles models, e.g. finite element (FE) models, are widely used. In many cases, modally reduced order models are applied, constructed from a limited number of structural modes. When proportional damping is assumed, the continuous-time decoupled equations of motion for modally reduced order models are given by

$$\ddot{\mathbf{z}}(t)+\boldsymbol{\Gamma}\dot{\mathbf{z}}(t)+\boldsymbol{\Omega}^2\mathbf{z}(t)=\boldsymbol{\Phi}^{\mathrm{T}}\mathbf{S}_{\mathrm{p}}(t)\mathbf{p}(t) \tag{1}$$

where $\mathbf{z}(t)\in\mathbb{R}^{n_{\mathrm{m}}}$ is the vector of modal coordinates, with n_{m} being the number of modes taken into account in the model. The excitation force is written as the product of a selection matrix $\mathbf{S}_{\mathrm{p}}(t)\in\mathbb{R}^{n_{\mathrm{dof}}\times n_{\mathrm{p}}}$, specifying the force locations, and a time history vector $\mathbf{p}(t)\in\mathbb{R}^{n_{\mathrm{p}}}$, with n_{p} being the number of forces. For the remainder of this paper, the selection matrix $\mathbf{S}_{\mathrm{p}}(t)$ is assumed to be time-invariant. The results, however, can be readily extended to the case where $\mathbf{S}_{\mathrm{p}}(t)$ is varying with time. The number of degrees of freedom is denoted by n_{dof}. $\boldsymbol{\Gamma}\in\mathbb{R}^{n_{\mathrm{m}}\times n_{\mathrm{m}}}$ is a diagonal matrix containing the terms $2\xi_j\omega_j$ on its diagonal, where ω_j and ξ_j are the natural frequency and modal damping ratio corresponding to mode j, respectively. $\boldsymbol{\Omega}\in\mathbb{R}^{n_{\mathrm{m}}\times n_{\mathrm{m}}}$ is a diagonal matrix as well, containing the natural frequencies ω_{j} on its diagonal, and $\boldsymbol{\Phi}\in\mathbb{R}^{n_{\mathrm{dof}}\times n_{\mathrm{m}}}$ is a matrix containing the mass normalized mode shapes $\boldsymbol{\phi}_j$ as columns. Throughout the paper it is assumed that the system does not contain rigid body modes, corresponding to a natural frequency $\omega_j=0$ rad/s.

The decoupled governing equations can be written in a state-space form, which after time discretization reads

$$\mathbf{x}_{[k+1]}=\mathbf{A}\mathbf{x}_{[k]}+\mathbf{B}\mathbf{p}_{[k]} \tag{2}$$

where $\mathbf{x}_{[k]}=\mathbf{x}(k\Delta t)$ and $\mathbf{p}_{[k]}=\mathbf{x}(k\Delta t)$, $k=1,\ldots,N$, Δt is the sampling time step, and N is the total number of samples. The state vector $\mathbf{x}_{[k]}$ consists of the modal displacements and velocities:

$$\mathbf{x}_{[k]}=\begin{bmatrix}\mathbf{z}_{[k]}\\ \dot{\mathbf{z}}_{[k]}\end{bmatrix} \tag{3}$$

The specific form of $\mathbf{A}$ and $\mathbf{B}$ depends on the time discretization scheme and will not be further considered. The reader is referred to [22] for a detailed overview of common time discretization schemes. As an alternative to models based on first principles, models can be directly identified from experimental vibration data using system identification techniques, see e.g. [23].

The output vector is generally written as

$$\mathbf{d}(t)=\mathbf{S}_{\mathrm{d,a}}\boldsymbol{\Phi}\ddot{\mathbf{z}}(t)+\mathbf{S}_{\mathrm{d,v}}\boldsymbol{\Phi}\dot{\mathbf{z}}(t)+\mathbf{S}_{\mathrm{d,d}}\boldsymbol{\Phi}\mathbf{z}(t) \tag{4}$$

where $\mathbf{S}_{\mathrm{d,a}}$, $\mathbf{S}_{\mathrm{d,v}}$, and $\mathbf{S}_{\mathrm{d,d}}\in\mathbb{R}^{n_{\mathrm{d}}\times n_{\mathrm{dof}}}$ are selection matrices indicating the degrees of freedom corresponding to the acceleration, velocity and displacement or strain measurements, respectively. The output vector is composed of $n_{\mathrm{d,d}}$ displacement or strain measurements, $n_{\mathrm{d,v}}$ velocity measurements and $n_{\mathrm{d,a}}$ acceleration measurements, where n_{d} is the sum of $n_{\mathrm{d,d}}$, $n_{\mathrm{d,v}}$, and $n_{\mathrm{d,a}}$.

Eq. (4) is transformed into its state-space form, using Eq. (1)

$$\mathbf{d}_{[k]}=\mathbf{G}\mathbf{x}_{[k]}+\mathbf{J}\mathbf{p}_{[k]} \tag{5}$$

The expressions for the state-output matrix $\mathbf{G}$ and the direct transmission matrix $\mathbf{J}$ do in general not depend on the time discretization scheme, because Eqs. (4) and (5) do not involve a time lag. The expressions for $\mathbf{G}$ and $\mathbf{J}$ are given by

$$\mathbf{G}=[\mathbf{S}_{\mathrm{d,d}}\boldsymbol{\Phi}-\mathbf{S}_{\mathrm{d,a}}\boldsymbol{\Phi}\boldsymbol{\Omega}^2\quad \mathbf{S}_{\mathrm{d,v}}\boldsymbol{\Phi}-\mathbf{S}_{\mathrm{d,a}}\boldsymbol{\Phi}\boldsymbol{\Gamma}],\quad \mathbf{J}=[\mathbf{S}_{\mathrm{d,a}}\boldsymbol{\Phi}\boldsymbol{\Phi}^{\mathrm{T}}\mathbf{S}_{\mathrm{p}}] \tag{6}$$

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