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Ranking paths in statistical energy analysis models with non-deterministic loss factors

Àngels Aragonès, Oriol Guasch *

GTM – Grup de Recerca en Tecnologies Mèdia, La Salle, Universitat Ramon Llull C/Quatre Camins 2, 08022 Barcelona, Catalonia, Spain

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ABSTRACT

Finding the contributions of transmission paths in statistical energy analysis (SEA) models has become an established valuable tool to detect and remedy vibro-acoustic problems. Paths are identified in SEA according to Craik's definition and recently, very efficient methods have been derived to rank them in the framework of graph theory. However, up to date classification schemes have only considered the mean values of loss factors for path comparison, their variance being ignored. This can result in significant errors in the final results. In this work it is proposed to address this problem by defining stochastic biparametric SEA graphs whose edges are assigned both, mean and variance values. Paths between subsystems are then compared according to a proposed cost function that accounts for the stochastic nature of loss factors. For an efficacious ranking of paths, the stochastic SEA graph is converted to an extended deterministic SEA graph where fast classification deterministic algorithms can be applied. The importance of nonneglecting the influence of the variance in path ranking is illustrated by means of some academic numerical examples.

1. Introduction

Transmission path analysis in statistical energy analysis (SEA) was introduced by Craik [1,2]. It often happens in SEA, that the energy reaching a target subsystem from a source subsystem, where external energy is being input, is mostly driven by a sometimes large but limited set of dominant transmission paths. Consequently, finding and classifying this set of paths becomes of interest to identify and remedy possible high frequency vibro-acoustic problems in structures. As noted in [3], up to date path classification had mostly relied on experience and intuition due to the prohibitive computational cost of the somehow naive approach of straightforward computing and then sorting paths, though some useful clues could be glanced from the analysis of groups of paths in [4] (see also [2]). Yet, the possibility to rank contributions of individual paths in an efficient way has been made possible recently by establishing a connection between SEA and graph theory [5]. In [6], the deterministic MPS algorithm [7] based on deviation path computations was adapted for this purpose (see also [8] for related applications of graph theory to SEA).

That said however, SEA does not deal with the dynamic response of a single structure but with that of an ensemble average of structures having similar but randomly varying parameters. In fact, the importance to quantify not only the mean energy per subsystem but also its variance, was highlighted from the very beginnings of SEA [9]. Since then, both

* Corresponding author. Tel.: +34 932902476; fax: +34 932902470.
E-mail addresses: oguasch@salle.url.edu, oguaschf@gmail.com (O. Guasch).

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nonparametric and parametric approaches have been followed to deal with uncertainties. Recently, some closed formulas have been derived for the former based on the Gaussian orthogonal ensemble [10,11] (see also the work in [12]). Analytic [3] as well as parametric methods to evaluate subsystem energy variability due to loss factor and input power uncertainties have been also developed [13]. However, in what concerns transmission paths, and to the best of the authors knowledge, no existent classification scheme has yet considered the influence of the statistical nature of SEA on path ranking. The need of doing so was pointed out in [14], where standard error propagation formulas were used to analyze the variability of paths, though this obviously not resulted in a unique list of sorted paths. Similarly, resorting to computationally expensive Monte Carlo simulations introducing variations on the loss factors of paths would neither provide a list of energy transmission paths (again only some information on the variability of path ranking could be attempted). Alternatively, a better option would be to make use of the Hurwicz criterion [15] to find a balanced solution between the best and worst scenarios in path analysis, though once again, this does not result in a single ranking of paths. It is precisely the aim of this paper to provide a classification methodology that results in a unique set of sorted paths which not only takes into account the mean values of the loss factors involved in them, but also their variances. Though academic in nature, it is the authors believe that providing a possible solution to such a problem is worth exploring and could be useful to the vibroacoustic engineer decision-making process. The proposed solution follows some of the suggestions in [16] to tackle with the general problem of path classification with random weights in the framework of graph theory.

The strategy exposed in this work to rank paths with non-deterministic loss factors involves several steps. According to Craik's definition [1], a first order path from a source subsystem to a neighbor receiver subsystem is defined by the ratio between the coupling loss factor from the source to the receiver, and the total loss factor of the receiver. A nth order path between two arbitrary subsystems in a SEA model is built from the consecutive products of n first order paths that link the intermediate subsystems one to another. Therefore, the initial step to deal with non-deterministic paths involves gaining information on the probability density functions (pdfs) that describe the loss factors involved in a first order path. Most available information at present concerns coupling loss factor pdfs for simple element connections involving springs [17], beams or plates [18–22], but little information is found e.g., on internal loss factors. As a consequence and as it will be shown later on, some simplifying hypotheses are to be made for them, as well as for combining loss factor pdfs in Craik's path formula. Once it is known how to assign variances to first order paths, the following step consists in building a biparametric stochastic SEA graph in analogy to what is done for the deterministic case [5,6]. The basic difference is that now two values (mean and variance) are assigned to each edge of the SEA graph instead of one. Next, in order to compare path weights for classification, one can no longer solely rely on their mean values but the variance information has to be incorporated somehow. As it will be shown, this can be done by defining a nonlinear cost function that combines mean and variance path values. Having this done, the final and most intricate step, consists in establishing an algorithmic strategy for the efficient comparison and classification of paths. The MPS algorithm used in [6] cannot be directly applied to the problem because stochastic graphs do not satisfy the optimality principle (i.e., a maximal transmitting path is not made of maximal transmitting subpaths, see [6,7] and references therein). To circumvent this difficulty it is proposed to resort to the procedure in [23], which consists in transforming a stochastic biparametric graph into an extended uniparametric graph. This will allow applying the very efficient MPS algorithm to the extended SEA graph and to classify energy transmission paths with non-deterministic loss factors. A preliminary version of some of the herein exposed results was recently presented in [24].

The paper is organized as follows. In Section 2, the notion of stochastic SEA graph is introduced and it is shown how to weight it using information from loss factor pdfs. The nonlinear cost function to compare paths is presented and the classification problem to be solved is mathematically posed. The strategy and algorithms used to rank non-deterministic paths are described in Section 3. Section 4 contains two benchmark examples where the algorithm is applied and the results become compared with those from the pure deterministic classification scheme. Conclusions close the paper in Section 5.

2. Problem statement

2.1. Deterministic and stochastic SEA graphs

SEA systems are often represented by means of block diagrams [9,2] like the one depicted in Fig. 1(a) for a four subsystem model. There are two kinds of elements in a SEA diagram, the blocks that correspond to SEA subsystems and the arrows, which may be of three different types: directed arrows connecting pairs of subsystems and symbolizing the power flow between them, non-connecting arrows pointing to subsystems and representing external input power, and non-connecting arrows leaving subsystems and standing for internally dissipated power. For instance, in Fig. 1(a) the arrow from A to C stands for the power flow W_{AC}, whereas W_A is the power injected to A and W_{Cd} the power dissipated in C.

A SEA block diagram clearly resembles a graph. In a nutshell, a graph G is an ordered pair $G=(U,E)$ consisting of a set of nodes U and a set of arcs E (also denoted edges). Each arc $e \in E$ establishes a connectivity relation between two nodes and it is defined by its tail u_i and head u_j nodes in U, i.e. $e=(u_i,u_j)$. Thus, for every SEA system one can define a SEA graph [5], $G_{SEA}=(U_{SEA},E_{SEA})$ such that every node u_i in U_{SEA} corresponds to a SEA subsystem and such that directed arcs $(u_i,u_j),(u_j,u_i) \in E_{SEA}$ exist between subsystems u_i and u_j, whenever they are coupled in the SEA model. Each arc (u_i,u_j) can be assigned a weight w_{ij}, which can be gathered in a weighting matrix $\mathcal{W}$ with elements $\mathcal{W}(i,j)=w_{ij}$. As an example, Fig. 1(b) presents the SEA graph associated to the SEA system in Fig. 1(a). Connecting SEA with graph theory allows to infer