Mechanical Systems and Signal Processing 41 (2013) 86–97

Contents lists available at ScienceDirect

Mechanical Systems and Signal Processing

journal homepage: www.elsevier.com/locate/ymssp

Importance analysis for models with correlated input variables using state dependent parameters approach

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A R T I C L E I N F O

Article history:
Received 15 December 2011
Received in revised form
18 March 2013
Accepted 2 August 2013
Available online 22 August 2013

Keywords:
Correlated input variable
Importance measure
Variance contribution
Uncorrelated contribution
Correlated contribution
State dependent parameters method

A B S T R A C T

For clearly exploring the origin of the variance of the output response in case the correlated input variables involved and establishing efficient method to calculate the importance measures of correlated input variables, a novel method on the state dependent parameters (SDPs) approach is proposed to decompose the contribution by correlated input variables to the output variance into two parts: the uncorrelated contribution due to the unique variations of a variable and the correlated one due to the variations of a variable correlated with other variables. In the proposed method, the transformation of the correlated inputs into independent and orthogonal ones and the calculation of the importance measures of the transformed independent ones are obtained by the SDP method simultaneously, thus it can improve the computational efficiency considerably in case of acceptable accuracy. In addition, the relationship between the existing independent orthogonalisation-based and the regression-based importance measures of the correlated input variables is revealed in the paper, which is then demonstrated by the numerical examples. The proposed method not only possesses higher computational efficiency in case of acceptable precision, but also has wider applicability compared with the polynomial chaos expansion (PCE) based method. Several numerical and engineering examples are used to demonstrate the advantages of the proposed method.

1. Introduction

Sensitivity analysis (SA), especially global SA, is widely used in engineering design and probability safety assessment. Indicators created for global SA purposes are called global uncertainty importance measures, and it is defined as how uncertainty in the output can be apportioned to different sources of uncertainty in the model input [1]. Global SA techniques, i.e. importance analysis techniques, aim at determining which of the input parameters influence output the most when uncertainty in the parameters is propagated through the model, and they have been fully developed in the last two decades. At present, many importance analysis techniques are available, such as nonparametric techniques [2–4], variance-based importance measure indices [5–8], and moment-independent importance measures [9–11]. Recently, a kind of derivative based global sensitivity measures has become popular due to computational efficiency and their link with the variance-based measures [12–14]. Among these importance analysis methods, variance-based importance measures have a quite general applicability since they can reflect the effect of the input variable on the output response simply and effectively.

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http://dx.doi.org/10.1016/j.ymssp.2013.08.001

However, most of the existing importance analysis techniques assume input variables independence, and a few studies have focused on importance analysis of the correlated input variables, which is usually the common case in engineering. The correlation of the input variables may affect the importance ranking dramatically, therefore, only the importance analysis methods taking the actual correlation into consideration can reflect the effect of the input variables on the output response reasonably and correctly. By now, some studies have been conducted to obtain the importance measures of correlated inputs [15–19], especially the method in Ref. [19], it can obtain the first order and total sensitivity indices of the correlated variables by a direct extension of the widely used Sobol' s sensitivity indices. However, all of these methods only provide an overall importance measure of one input variable, which does not distinguish the correlated or uncorrelated contribution of one input variable. For response models with correlated input variables, to explore the origin of the uncertainty of the output response clearly, the contribution of uncertainty to output response by an individual input variable should be divided into two parts: the uncorrelated contribution (by the uncorrelated variations, i.e. the unique variations of a variable which cannot be explained by any other parameters) and correlated contribution (by the correlated variations, i.e. variations of a variable which are correlated with other input variables) [20]. As pointed out in Ref. [20] that the distinction between uncorrelated and correlated contributions of uncertainty for an individual variable is very important, since it can help engineers decide if they need to focus on the correlated variations among specific variable (if the correlated contribution dominates) or the variable itself (if the uncorrelated contribution dominates). Based on this idea, a regression-based method is proposed in Ref. [20] to decompose the total variance of the output response into partial variances contributed by the correlated and uncorrelated variations of the input variables. However, this method is only suitable for the case where the relationship between output response and input variables is linear or approximately linear. A more robust and similar treatment for correlated input variables is proposed in Ref. [21] where the total contribution of an input variable or a subset of input variables to the variance of the output response is decomposed into structural contribution (reflecting the system structure) and correlative (reflecting the correlated input probability distribution) one. This treatment can deal with both linear and nonlinear response function. However, when decomposing the contribution of correlated input variables, both the methods in Refs. [20,21] do not consider the effects of the interaction among variables. Therefore, they cannot analyze the importance of the correlated input variables completely.

To overcome the limitations in Refs. [20,21], a set of variance-based importance measures is proposed in Ref. [22] to perform importance analysis of models with correlated inputs. The definition of those measures is based on a specific orthogonalisation of the inputs and ANOVA-representations of the model output. They can not only support nonlinear models and nonlinear dependencies, but also consider the effect of the interaction among variables when analyzing the importance of the correlated input variables. However, Ref. [22] does not use a unified method to perform the two essential steps for the importance analysis of the correlated variables, i.e. independent orthogonalisation of the correlated inputs and calculation of the importance measures of the transformed independent variables. Additionally, the polynomial chaos expansion (PCE) method they employed is restricted by the form of the polynomial basis, the dimensionality of the input variables and the computational cost etc. In this paper, a novel method on the state dependent parameters (SDP) approach is proposed to decompose the contribution by correlated input variables to the variance of output response. In the proposed method, the independent orthogonalisation of the correlated inputs and the calculation of the corresponding importance measures are performed by the unified SDP-based method, which can save computational cost considerably. The proposed method can not only improve the computational efficiency considerably in case of acceptable accuracy, but also overcome the limitations of the PCE method used in Ref. [22]. Therefore, it can provide an effective tool for the importance analysis of the correlated input variables. In addition, the relationship between the existing independent orthogonalisation-based and the regression-based importance measures of the correlated inputs is revealed in the paper. Two numerical examples with available analytical solutions are then used to demonstrate this relationship, and two engineering examples are used to show the advantages of the proposed method.

2. Variance-based importance measures of the correlated input variables and their relationship

2.1. Regression-based importance measures of the correlated input variables

For response models with correlated input variables, the uncertainty contribution of an individual variable not only consists of the uncorrelated contribution resulting from the variation of the variable itself, but also contains the correlated contribution. Based on this idea, the uncertainty contribution η_i of the correlated input variables to the output response is decomposed into uncorrelated contribution η_i^u and correlated one η_i^c in Ref. [20], i.e.

$$\eta_i = \eta_i^u + \eta_i^c \tag{1}$$

By exploiting regression-based method to regress y over different parts of the input variables, the uncorrelated and correlated contributions by an individual variable to the variance of the model output can be derived [20]. This method can successfully measure the uncertainty contribution when the relationship between output response and input variables is linear or approximately linear. However, when nonlinearity is present in the model, it cannot work well any more.