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# Families of moment matching based, structure preserving approximations for linear port Hamiltonian systems☆

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ABSTRACT

In this paper we propose a solution to the problem of moment matching with preservation of the port Hamiltonian structure, in the framework of time-domain moment matching. We characterize several families of parameterized port Hamiltonian models that match the moments of a given port Hamiltonian system, at a set of finite interpolation points. We also discuss the problem of Markov parameters matching for linear systems as a moment matching problem for descriptor representations associated with the given system, at zero interpolation points. Solving this problem yields families of parameterized reduced order models that achieve Markov parameter matching. Finally, we apply these results to the port Hamiltonian case, resulting in families of parameterized reduced order port Hamiltonian approximations.



## 1. Introduction

Port Hamiltonian systems represent an important class of systems used in modelling, analysis and control of physical systems; see e.g. Ortega, van der Schaft, Maschke, and Escobar (2002) and van der Schaft (2000). These representations are used in lumped parameter system analysis and control stemming from, e.g., mechanical systems, electrical systems, electromechanical systems, and power systems. However, physical modelling often leads to (port Hamiltonian) systems of high dimension, usually difficult to analyse and simulate and unsuitable for control design.

In the problem of model reduction moment matching techniques represent an efficient tool; see e.g. Antoulas (2005), Antoulas and Sorensen (1999), Byrnes and Lindquist (2008), Feldman and Freund (1995), Grimme (1997), Jaimoukha and Kasenally (1997) and van Dooren (1995) for a complete overview for linear systems. Using a numerical approach based on Krylov projection methods the (reduced order) model is obtained by efficiently constructing a lower degree rational function that approximates a given transfer function (assumed rational). The low degree rational function matches the given transfer function at various points in the complex plane.

Recently, in Wolf, Lohmann, Eid, and Kotyczka (2010), the rational interpolation problem for linear port Hamiltonian systems has been addressed using Krylov projection methods, yielding reduced order models that match the moments of the given port Hamiltonian system at a set of prescribed finite or infinite interpolation points. Improved procedures for MIMO systems have been developed in Gugercin, Polyuga, Beattie, and van der Schaft (2012) and Polyuga and van der Schaft (2010), where a near-optimal port Hamiltonian approximation that satisfies a set of tangential interpolation conditions is proposed. Furthermore, in Polyuga and van der Schaft (2009, 2010) the partial realization problem for port Hamiltonian systems has been considered.

In this paper we study the problem of computing low order approximations that match a set of prescribed moments at a set of finite or infinite points, of a given SISO, port Hamiltonian, linear system and preserve the port Hamiltonian structure, in the recent framework of time-domain moment matching, introduced in Astolfi (2010a), described in the following problem.

**The General Approximation Problem (GAP).** Given a linear, port Hamiltonian, SISO system,

(1) compute *the families* of *parameterized* reduced order models that match the moments of a given port Hamiltonian system

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at a set of finite or infinite interpolation points (in this case the models match a set of prescribed Markov parameters);

(2) characterize the families of reduced order models which satisfy the following properties: the models match the moments of the given port Hamiltonian system and *preserve the port Hamiltonian structure*. In other words, from the classes of models that achieve moment matching we find the reduced order models that inherit the port Hamiltonian form. □

In the case of matching at finite interpolation points we obtain families of parameterized *state-space*, reduced order port Hamiltonian models that approximate the given port Hamiltonian system. All the reduced order state-space models share the same transfer function. In the SISO case, the state space parameters are used to enforce additional structure constraints such as diagonal Hamiltonian function, diagonal dissipation, etc. However, at the moment we are not able to determine which of the original variables and their meaning is retained in the reduced order model. In the MIMO case the free parameters can be used to define appropriate directions such that the reduced order models satisfy prescribed sets of tangential interpolation conditions. Regarding the computational aspect, we establish connections between the models from the families of *parameterized* port Hamiltonian reduced order models that achieve moment matching and the counterpart approximations obtained using Krylov projections, i.e., there is a relation between the free parameter and the Krylov projector.

We study the problem of Markov parameters matching, i.e., the partial realization problem, by extending the time-domain moment matching results to the case of interpolation points at infinity. We define the notion of moment for a class of linear, descriptor representations, associated with the transfer function of the given system, in terms of the (unique) solutions of *generalized* Sylvester equations and their dual counterparts. In particular, the Markov parameters of a given system are the moments of the associated descriptor realization at zero. Furthermore, we relate the moments to the steady-state response, provided that it exists, of the descriptor realization driven by/driving signal generators. Performing model reduction we obtain several families of parameterized, (descriptor) reduced order models that match a set of prescribed moments of the descriptor realization associated with the transfer function of a given linear system. In particular, matching at zero yields classes of reduced order models that match the Markov parameters of the given linear system. As in the rational interpolation case, we establish relations between the *parameter* which define the family of the port Hamiltonian approximation that matches a set of Markov parameters and a Krylov projector, giving insight into the computational issue of the proposed solution.

Finally, we apply these results to linear port Hamiltonian systems, resulting in *families* parameterized of state-space, reduced order port Hamiltonian models that match the Markov parameters of the given port Hamiltonian system. Note that the free parameters can be used to enforce additional physical structure to the approximant. We mention that, to our knowledge, there is no structured procedure to determine the number of interpolation points needed. However, based on Ionescu, Scherpen, Iftime, and Astolfi (2012) the number of interpolation points can be related to the order of the dynamics associated with the higher order Hankel singular values of the given system. To conclude, we mention that the scope of the paper is not to address computational issues, but to propose a system-theoretical based framework for port Hamiltonian reduced order modelling, consistent with the existing theory and suitable for future *nonlinear* extension.

The paper is organized as follows. In Section 2 we give a brief overview of the definition of moments and moment matching for linear port Hamiltonian systems, as well as of the family of parameterized reduced order models that achieve moment matching at a set of finite interpolation points. We also recall the procedures to obtain a port Hamiltonian approximation using Krylov projections for both SISO and MIMO systems. We present existing choices of interpolation points which yield accurate (in some sense) approximations. The section is completed with the formulation of the model reduction problem to be solved, a particular case of the (GAP) for matching at finite interpolation points. In Section 3 we study the general problem of time-domain moment matching for a class of descriptor representations associated with the transfer function of a given linear system. In particular we show that matching the moments of the descriptor realization at zero is equivalent to matching the Markov parameters of the given system. Moreover, we obtain the classes of reduced order models that match a set of the Markov parameters of the given system. These models are then related to their Krylov projection counterparts. Based on Section 2, in Section 4 we discuss the problem of moment matching at a set of finite interpolation points, with preservation of the port Hamiltonian structure, and characterize the port Hamiltonian reduced order models. Furthermore, we give a necessary and sufficient condition for a reduced order model that achieves moment matching to be a port Hamiltonian model. We describe the families of parameterized state-space port Hamiltonian approximations, the free parameters of which can be used to enforce further properties. We also present a procedure that allows the computation of such a family of models. Based on the results from Section 3, in Section 4.2 we obtain the classes of reduced order, port Hamiltonian models that match the Markov parameters of a given port Hamiltonian system, proposing a procedure that allows the computation of such families of models. In Section 5, we give an example that illustrates the results. The paper is completed by a Conclusions Section.

Preliminary results have been presented in Ionescu and Astolfi (2011). This paper constitutes a complete version of the aforementioned conference paper, with results presented, discussed and proven in detail. Furthermore, in this paper we develop new results based on the solution of the dual Sylvester equation resulting in an extended system-theoretic characterization of the moment matching for port Hamiltonian systems. In addition, a discussion of the multiple-input multiple-output case is given. Moreover, connections with previous work and results from the literature, especially Krylov projection-based modelling, are established and examples are given to illustrate the theory.

*Notation.* $\mathbb{R}$ is the set of real numbers and $\mathbb{C}$ is the set of complex numbers. $\mathbb{C}^0$ is the set of complex numbers with zero real part and $\mathbb{C}^-$ denotes the set of complex numbers with negative real part. $A^* \in \mathbb{C}^{n\times m}$ denotes the transpose and complex conjugate of the matrix $A \in \mathbb{C}^{m\times n}$. If $A$ is a real matrix, then $A^*$ is the transpose of $A$. $\sigma(A)$ denotes the set of eigenvalues of the matrix $A$ and $\emptyset$ denotes the empty set. $0_{n\times\nu} \in \mathbb{C}^{n\times\nu}$ is the matrix with all elements equal to 0. If $n = 1$, then $0_{1\times\nu} = [0, \ldots, 0] \in \mathbb{R}^{1\times\nu}$ and $0_\nu = 0^*_{1\times\nu}$. $\delta(t)$ denotes the Dirac $\delta$-function. The triple $(A, B, C)$ denotes the linear, time-invariant system described by the equations $\dot{x} = Ax+Bu$, $y = Cx$, with input $u(t)$ from a well defined set of inputs, output $y(t)$ and state $x(t)$.

## 2. Preliminaries

Let $J \in \mathbb{R}^{n\times n}$ be a skew symmetric matrix and $R \in \mathbb{R}^{n\times n}$, $Q \in \mathbb{R}^{n\times n}$ be two symmetric matrices. Consider the single-input, single-output, minimal, port Hamiltonian system

$$\dot{x} = (J - R)Qx + Bu, \qquad y = B^*Qx, \tag{1}$$

where $x(t) \in \mathbb{R}^n$, $u(t) \in \mathbb{R}$, $y(t) \in \mathbb{R}$ and $B \in \mathbb{R}^n$. The Hamiltonian is $\mathcal{H}(x) = \frac{1}{2}x^*Qx$. The transfer function of system (1) is given by $K : \mathbb{C} \to \mathbb{C}$, $K(s) = B^*Q(sI - (J - R)Q)^{-1}B$. Let $s_i \in \mathbb{C}$, $i = 1, \ldots, \nu$ be such that $s_i \notin \sigma((J - R)Q)$. The moments of (1) at