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Sensor placement methods for an improved force identification in state space

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ABSTRACT

The accuracy and effectiveness of force identification in time domain based on state space can be influenced by the conditioning of the structural system Markov parameter matrix. Two different sensor placement methods based on the conditioning analysis of the Markov parameter matrix for improving the identification of input force are presented in this paper. The first one is based on direct computation of the condition number of the matrix, and it would involve computation for many different combinations of candidate sensor locations. It would be time consuming particularly when a large number of candidate combinations of sensor locations is considered. The second approach is based on the correlation analysis of the system Markov parameter matrix. A sensor correlation matrix is defined and the correlation criterion, which can indicate the ill-conditioning of the Markov parameter matrix, is introduced. The performances of these two methods are compared in numerical simulations with respect to their efficiency and accuracy. It is concluded that the performance of both methods is similar when the number of candidate combination of sensors is small. However, when there are many candidate combinations of sensor locations, the method based on correlation analysis of the Markov parameter matrix performs better with consistently good sensor placement for force identification and much less computation effort.

1. Introduction

Knowledge of the external force on a structural system is of great importance in many structural dynamic problems. However, its direct measurement with transducer is a difficult task when the locations cannot be accessed, while vibration responses can be conveniently measured. This partly explained why indirect methods are often preferred to direct measurement, in which forces can be identified based on the inverse structural analysis. Various indirect force identification methods have been proposed in recent years. Techniques involved are either in the frequency domain or in the time domain. Fourier Transform is often used in the frequency domain methods to establish the relationship between the force and response via the frequency response function [1,2]. However, it is well known that the identification process requires the inversion of the transfer function, and this suffers from inherent instability caused by the ill-posed inverse problem.

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In particular, measurements containing noise would have an adverse combination effect with the ill-posed problem leading to computation instability.

The usual approach to the solution of an ill-posed problem is to transform it into a well-posed problem by using additional information on the solution sought. The methods such as singular value decomposition (SVD) and Tikhonov regularization method [3–5] have been used to solve the ill-posed problem. There were also developments in time domain force identification. Kammer [6] presented a time domain method for estimating the discrete input force based on the measured response and the structural system Markov parameters. The inverse system Markov parameters can be computed from the forward Markov parameters using a linear prediction algorithm. Mao et al. [7] proposed a time domain force identification approach for a linear system based on the Markov parameters with precise computation and regularization technique. Law and Ding [8] developed a time domain force identification method for a substructure. The interface forces as well as the external forces acting on the substructure can be identified through the Tikhonov regularization method in state space.

The accuracy of the inverse analysis may vary significantly with different spatial location of the response measurements, and several approaches have been developed for choosing the sensor locations. Blau [9] suggested a frequency dependent criterion for locating sensors in the force spectra identification. Thite and Thompson [10] commented that this technique is difficult to apply to the whole frequency range of interest, and they proposed a method to select the sensor locations to improve the inverse force determination based on a composite condition number. The minimum composite condition number of the transfer function matrix associates with a reduction in the ill-conditioning of the transfer function as well as in the error of the reconstructed forces.

There are also other methods on the selection of sensor locations in modal testing and condition monitoring of structures. Kammer [11] proposed an Effective Independence (EFI) method to quantify the contributions of response measurements so that the modal states of targeted modes can be optimally observed. Lim [12] employed the generalized Hankel matrix, a function of the system controllability and observability, to develop an approach which can determine sensor locations based on a given rank for the system observability matrix while satisfying modal test constraints. Hemez and Farhat [13] proposed an Energy Matrix Rank Optimization method which is to maximize the strain energy information in instrumented structural members. This method requires both the target mode shapes for the candidate sensors as well as the structural stiffness. Heo et al. [14] derived a Kinetic Energy Optimization Technique (EOT) with the formulation similar to EFI, and the difference lies in the quantity that is optimized. The EFI method maximizes the Fisher′s information matrix while the EOT optimizes the kinetic energy matrix. A limitation of the EFI method is that the sensor locations with low energy content may be selected with a consequent possible loss of information. The EFI-DPR (Driving Point Residue) method eliminates this problem by multiplying the candidate sensor contribution of the EFI method with the corresponding DPR coefficient [15]. Papadimitriou et al. [16] introduced the information entropy norm as a measure that best corresponds to the objective of structural testing which is to minimize the uncertainty in the model parameter estimates. The spatial correlation is important to avoid redundant information provided by neighboring sensors with distance less than the characteristic length of the highest contributing mode. Valuable insight was provided into the effect of spatial correlation of the prediction error on the optimal placement of sensors for modal identification or parameter estimation in finite element model updating problems encountered in structural dynamics [17].

The accuracy and effectiveness of force identification in time domain based on state space can be influenced by the conditioning of the structural system Markov parameter matrix which may vary significantly with measurement location. In this paper, two different sensor placement methods based on the conditioning analysis of Markov parameter matrix for improving the identification of input force are presented. The first one is based on direct computation of the condition number of the Markov parameter matrix, and it may involve computation with many different combinations of candidate sensor locations. It would be time consuming particularly when a large number of candidate combinations of sensor locations is considered. The second approach is based on the correlation analysis of the Markov parameter matrix. The auto-correlation factor and cross-correlation factor are introduced in a sensor correlation matrix. A sensor correlation criterion, which can be used as a measure to select the sensor locations, is defined. A set of responses can be found to associate with a minimum sensor correlation criterion. Numerical simulations with a plane truss structure and a three-dimensional frame structure are used to validate these methods and compare their performances with respect to their efficiency and accuracy with noisy measurements.

2. Force identification algorithm in state space

2.1. State space equation of motion

For a general finite element model of a linear damped elastic structure, the equation of motion can be written as,

$$M\ddot{x} + C\dot{x} + Kx = f \tag{1}$$

where matrices M, C, and K are the mass, damping and stiffness matrices of the structural system, respectively. $\ddot{x}$, $\dot{x}$ and x are vectors of acceleration, velocity and displacement of the structural system, respectively. f is the vector of external excitation with matrix L mapping these forces to the associated degree-of-freedoms (DOFs) of the structure. Rayleigh damping