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Brief paper

Consensus of sampled-data multi-agent networking systems via model predictive control☆

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ABSTRACT

This paper presents a theoretical framework to design and analyze the consensus of a sampled-data multi-agent networking system via model predictive control (MPC). We introduce a distributed MPC weighted-average consensus protocol for a continuous-time multi-agent network in the sampled-data setting, and prove such a sampled-data multi-agent system asymptotically reaches the weighted-average consensus via the distributed MPC protocol with both fixed and switching network topologies. Numerical examples verify the effectiveness of the distributed MPC consensus protocol, which is beneficial to the convergence of consensus and the feasible range of the sampling interval.

1. Introduction

Collective behaviors of multiple autonomous agents, which vividly exist in nature such as flocking of birds and swarming of fishes, have been extensively studied by researchers from the fields of biology, computer science, automatic control and distributed computing (Ferrari-Trecate, Galbusera, Marciandi, & Scattolini, 2009; Jadbabaie, Lin, & Morse, 2003; Moreau, 2005; Olfati-Saber, 2006; Olfati-Saber, Fax, & Murray, 2007; Olfati-Saber & Murray, 2004; Qin, Gao, & Zheng, 2011; Ren & Beard, 2005; Ren, Beard, & Arkins, 2007; Reynolds, 1987; Tsitsiklis & Athans, 1984; Vicsek, Czirók, Ben-Jacob, Cohen, & Shochet, 1995; Xiao & Boyd, 2004; Yu, Chen, & Cao, 2010; Zhan & Li, 2013; Zhang, Chen, & Stan, 2011; Zhang, Chen, Stan, Zhou, & Jan, 2008; Zhang, Chen, Zhou, & Stan, 2008; Zhang & Tian, 2010). The underlying mechanism, that global coordinated behaviors arise from local interactions among agents, helps us gain a more accurate and deeper understanding of cooperative control of a multi-agent networking system. Due to high efficiency and operational capability, cooperative control has broad applications in many areas including formation control (Cao, Yu, & Anderson, 2011; Fax & Murray, 2004), congestion control in communication networks (Antoniou, Pitsillides, Blackwell, & Engelbrecht, 2009), and distributed sensor networks (Kar & Moura, 2010; Yu, Chen, Wang, & Yang, 2009). The study of cooperative control mainly focuses on consensus (Ferrari-Trecate et al., 2009; Jadbabaie et al., 2003; Olfati-Saber et al., 2007; Olfati-Saber & Murray, 2004; Qin et al., 2011; Ren & Beard, 2005; Ren et al., 2007; Tsitsiklis & Athans, 1984; Vicsek et al., 1995; Xiao & Boyd, 2004; Yu et al., 2010; Zhang et al., 2011; Zhang, Chen, Stan et al., 2008; Zhang, Chen, Zhou et al., 2008; Zhang & Tian, 2010), synchronization, and flocking (Olfati-Saber, 2006; Reynolds, 1987; Zhan & Li, 2013), among which, consensus is a fundamental and widely concerned problem.

Consensus means that all agents reach an agreement on certain quantities of interest, and the investigations on consensus have been devoted to designing protocols for a multi-agent system to reach an agreement and deriving sufficient conditions to guarantee the system convergence. In 1984, Tsitsiklis and Athans (1984) initialized a distributed algorithm for a group of agents to reach an agreement, and derived the conditions for the asymptotic convergence of each agent's decision sequence and the asymptotic consensus of all agents' decisions. Vicsek et al. proposed the famous Vicsek model in Vicsek et al. (1995), which considered the discrete-time consensus of agents' headings. In Jadbabaie et al. (2003), Jadbabaie et al. provided a theoretical proof for the emergence of alignment in the Vicsek model, firstly pointing out that a jointly connected collection of graphs along a time interval is a sufficient

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condition to guarantee consensus. The theoretical framework for posing and solving consensus problems of networked multi-agent systems was introduced by Olfati-Saber and Murray in Olfati-Saber and Murray (2004). Moreover, many latest results have been reported for the consensus of multi-agent systems with different features taken into account, such as time delays (Olfati-Saber & Murray, 2004; Qin et al., 2011), sampled data (Zhang & Tian, 2010), second-order dynamics (Qin et al., 2011; Yu et al., 2010), and predictive mechanisms (Ferrari-Trecate et al., 2009; Zhang et al., 2011; Zhang, Chen, Stan et al., 2008; Zhang, Chen, Zhou et al., 2008).

More related to the topic of this paper, Zhang, Chen, Stan et al. (2008) and Zhang, Chen, Zhou et al. (2008) developed a consensus protocol where a model predictive control (MPC) term is added to the routine consensus protocol (Olfati-Saber & Murray, 2004), and numerically verified the performance of consensus. They further assumed that only a few pinned agents were equipped with the predictive controllers, which required global information of the whole network, and proved the faster convergence speed in Zhang et al. (2011). Besides, Ferrari-Trecate et al. (2009) proposed the decentralized MPC consensus schemes with constraints on every agent's input, and exploited geometric properties of the optimal paths followed by individual agents as well as the consensus convergence, relying on the proofs in Moreau (2005) targeting the consensus in a network with time-varying communication links.

In this paper we focus on the consensus of a sampled-data multi-agent networking system via the model predictive control (MPC) method, and our main contributions include: (1) We present both the analytical solution to the distributed MPC consensus problem and the proof of asymptotical convergence to the consensus. (2) We consider a sampled-data multi-agent system and numerically explore the role of the sampling interval in reaching a consensus, while most of the previous related studies (Ferrari-Trecate et al., 2009; Zhang et al., 2011; Zhang, Chen, Stan et al., 2008; Zhang, Chen, Zhou et al., 2008) were based on discrete-time models. The rest of this paper is organized as follows: Section 2 introduces the concepts of consensus and the routine consensus protocols presented in Olfati-Saber and Murray (2004). In Section 3, we design a distributed MPC weighted-average consensus protocol for a sampled-data multi-agent networking system, and prove the asymptotical convergence of weighted-average consensus with both fixed and switching network topologies addressed. Section 4 verifies the performance of our proposed distributed MPC consensus protocol with numerical examples, and finally, Section 5 concludes the whole paper.

2. Preliminaries

We first introduce the mathematical notations to use throughout this paper. $\mathbb{R}^n$ denotes the set of n-dimensional real column vectors. $\mathbf{1} = (1, 1, \dots, 1)'$, $\mathbf{0} = (0, 0, \dots, 0)'$, and I is the identity matrix with an appropriate dimension if no confusion arises. $\|x\| = (x'x)^{1/2}$ is the 2-norm of a column vector x, and $\otimes$ denotes the Kronecker product.

A weighted graph is denoted by $\mathcal{G} = (\mathcal{V}, \mathcal{E}, A)$, consisting of a set of vertices $\mathcal{V} = \{1, 2, \dots, n\}$, edges $\mathcal{E} \subseteq \{(i,j) : i,j \in \mathcal{V}, j \neq i\}$, and a weighted adjacency matrix $A = [a_{ij}]$ with nonnegative adjacency elements a_{ij}. A graph $\mathcal{G}$ is undirected if $(i,j) \in \mathcal{E} \iff (j,i) \in \mathcal{E}$, and this paper is only concerned with undirected graphs. An undirected graph is connected if there exists a path, i.e., a sequence of distinct edges such that consecutive edges are joint, between any two vertices. The adjacency elements associated with the edges of a graph are positive, and the others are zeros. The neighbors of agent i are denoted by $N_i = \{j \in \mathcal{V} : (i,j) \in \mathcal{E}\}$.

Let $x_i \in \mathbb{R}$ and $x = (x_1, x_2, \dots, x_n)' \in \mathbb{R}^n$ denote the state of agent (node) i and the state of a multi-agent networking system, respectively. We say a network of agents reach a consensus if and only if $x_i = x_j$ for all $i, j \in \mathcal{V}, i \neq j$, where all agents reach the so-called consensus point α. The special case of $\alpha = Ave(x(0)) = 1/n \sum_{i=1}^{n} x_i(0)$ is average-consensus. Consider a network of continuous-time integrator agents with the dynamics described as follows

$$\dot{x}_i = u_i. \tag{1}$$

In Olfati-Saber and Murray (2004), Olfati-Saber and Murray designed the following protocol for the agents to achieve the average-consensus

$$u_i = \sum_{j \in N_i} a_{ij}(x_j - x_i). \tag{2}$$

In the general case of weighted-average consensus with a desired weighting vector $\gamma = (\gamma_1, \gamma_2, \dots, \gamma_n)'$, $\gamma_i > 0, \forall i \in \mathcal{V}$, the protocol is

$$u_i = \frac{1}{\gamma_i} \sum_{j \in N_i} a_{ij}(x_j - x_i), \tag{3}$$

and the weighted-average consensus point is

$$\alpha = \gamma' x(0)/(\mathbf{1}'\gamma). \tag{4}$$

Since average-consensus is a special case of weighted-average consensus when $\gamma_1 = \gamma_2 = \cdots = \gamma_n = 1$, we mainly discuss the weighted-average consensus of a multi-agent network in this paper.

3. Sampled-data consensus via distributed model predictive control

Rewrite Eq. (1) of agent i in the sampled-data setting as

$$\dot{x}_i(t) = u_i(t_k), \quad \text{if } t \in [t_k, t_{k+1}),\ k = 0, 1, \dots \tag{5}$$

where $\{t_k\}_{k=0}^{+\infty}$ are the discrete sampling instants, and $t_k = kT$ with the periodic sampling interval T. Discretizing (5) with the sampling interval T yields

$$x_i(k+1) = x_i(k) + Tu_i(k) \tag{6}$$

where we let k stand for t_k. Now we focus on designing a distributed MPC consensus protocol for a network of such agents to reach the weighted-average consensus point (4).

Consider a network of n agents following (5), each of which solely has the access to the states of its neighbors, and decompose the whole network into n subsystems accordingly. For agent i, its subsystem consists of itself and its neighbors, denoted by $\mathcal{S}_i = \{n_1^i, n_2^i, \dots, n_{|N_i|+1}^i\}$, $n_1^i < n_2^i < \cdots < n_{|N_i|+1}^i$, $n_j^i \in N_i \cup i, j = 1, 2, \dots, |N_i| + 1$. Agent i collects the state of $\mathcal{S}_i$ as $x^i = (x_{n_1^i}, x_{n_2^i}, \dots, x_{n_{|N_i|+1}^i})'$, and predicts the future state of $\mathcal{S}_i$ along the sampling instants according to

$$x^i(k+j+1|k) = x^i(k+j|k) + Tu^i(k+j|k), \quad j = 0, 1, \dots \tag{7}$$

with $(k+j|k)$ denoting the prediction of future instant $k+j$ at instant k, and $u^i = (u^i_{n_1^i}, u^i_{n_2^i}, \dots, u^i_{n_{|N_i|+1}^i})'$ denoting the control input for $\mathcal{S}_i$ decided by agent i. Note that the component $u_j^i, \forall j \in \mathcal{S}_i$, in u^i is the control input for agent j decided by agent i.

Based on Eq. (7) with the known $x^i(k)$, agent i optimizes the following MPC cost function to compute the control input u^i for $\mathcal{S}_i$ at instant k:

$$\min_{\{u^i(k+j|k)\}_{j=0}^{H_u-1}} J_i(k) = \min_{\{u^i(k+j|k)\}_{j=0}^{H_u-1}} \left(\sum_{j=1}^{H_p} \|c^i x^i(k+j|k)\|^2 + \lambda \sum_{j=0}^{H_u-1} \|u^i(k+j|k)\|^2 \right) \tag{8}$$