Mechanical Systems and Signal Processing 41 (2013) 613–638

Contents lists available at ScienceDirect

Mechanical Systems and Signal Processing

journal homepage: www.elsevier.com/locate/ymssp

Design sensitivity analysis of dynamic response of nonviscously damped systems

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ARTICLE INFO

Article history:
Received 12 November 2012
Received in revised form
12 May 2013
Accepted 13 August 2013
Available online 8 September 2013

Keywords:
Nonviscous damping
Sensitivity analysis
Viscoelastic structure
Dynamic analysis
Direct differentiation method
Adjoint variable method

ABSTRACT

The sensitivity problem of dynamic analysis of linear nonviscously damped systems is considered. The assumed nonviscous damping forces depend on the past history of motion via convolution integrals over some kernel functions. The nonviscous damping model can be alternatively chosen from familiar viscoelastically damping structures and is considered as a further generalization of the familiar viscous damping. The computations of dynamic responses are reviewed for the purpose of design sensitivity analysis development. The dynamic response can be easily calculated using direct frequency response method and modal superposition method when the dynamic equation of motion of nonviscously damped systems is transformed into the frequency domain using the Laplace transform. It is shown that the dynamic response of nonviscously damped systems can be obtained using traditional modal analysis in a familiar manner used in undamped or viscously damped systems. The discrete Fourier transform and inverse discrete Fourier transform algorithms are also suggested to obtain the displacement in the time domain. Based on these expressions of dynamic response, the adjoint variable and direct differentiation methods, originally presented to obtain the dynamic response sensitivity of undamped or viscously damped systems, are both developed for efficiently and accurately calculating the sensitivity of dynamic response of nonviscously damped systems. Finally, some case studies are used to show the application, effectiveness and some characters of the derived formulas. The numerical sensitivity results show the sensitivity obtained using the developed methods are in excellent agreement with the finite difference results. However, the finite difference method suffers from computational inefficiency and possible errors.

1. Introduction

Design sensitivity analysis of mechanical systems and engineering structures is used to quantify the relationship between parameters used to define an optimum design and calculate outputs used to measure their performance. A significant body of research has been devoted to the computation and application of design sensitivity analysis (see, e.g., van Keulen et al. [1], Haftka and Adelman [2], Haug et al. [3] or Choi and Kim [4]). Structural design sensitivity analysis with respect to structural design parameters has become an integral part of many engineering applications such as structural health monitoring, reliability, model updating, structural optimization, dynamic modification, and approximate reanalysis

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http://dx.doi.org/10.1016/j.ymssp.2013.08.009

techniques. Generally speaking, design sensitivity can be used to predict the changes of the system response with respect to the changes of design parameters, select a search direction for optimal design process, obtain an approximation model for iteration process, and assess the effects of uncertainties of structure property to system response. Mottershead et al. [5] pointed out that the sensitivity-based method is probably the most successful of the many methods to the problem of model updating.

Design sensitivity analysis of dynamic response has been studied by several authors. Zhang and Kiureghian [6] presented a finite element solution method to deal with the dynamic response sensitivity of inelastic structures subjected to transient loading. Benfratello et al. [7] presented an algorithm to evaluate the stochastic sensitivity of dynamic response in the time domain for stationary Gaussian and non-Gaussian white input processes. Kim and Choi [8] presented a method by directly deriving the unconditionally stable implicit numerical equation. It has been demonstrated that the analytical equation of the design sensitivity has the same left-hand side form as the governing dynamic equation. Hence it is possible to use the same factorized matrix of dynamic analysis equation. Bogomolni et al. [9] studied the design sensitivity analysis of discrete linear systems subjected to dynamic loading and presented an algorithm for reducing the number of differential equations needed to be solved during the solution process by means of the combined approximation approach (see Kirsch [10]). Recently, Liu et al. [11–14] studied the sensitivity analyses of structures subjected to transient loading or random excitation. Petrov [15] presented an effective method to calculate the sensitivity of the forced response of strongly nonlinear structures. Becker et al. [16–18] studied the Bayesian sensitivity analysis of dynamic response of nonlinear finite element models in the time domain.

The steady-state frequency response of mechanical and structural dynamic systems is of interest in dynamic design problems subjected to harmonically varying external loading that may be caused by a reciprocating power train or such other rotating machine parts (such as motors, rotating machinery, unbalanced tires, helicopter blades and forging hammers). Kramer and Grierson [19] obtained the response sensitivity by direct differentiation of the equations of motion by means of assuming harmonic loading and mode superposition. Explicit sensitivity formulas of performance measures of frequency response have been obtained using the adjoint variable and direct differentiation methods for viscously damped systems [20,21] and have been used for structural optimization (see, e.g., Choi and Lee [20]). Also, the explicit sensitivity formulas are available in some existing commercial codes (e.g., NASTRAN [22]). Ting [23] derived an iterative scheme to calculate frequency response sensitivities. Based on mode superposition, Qu [24–26] developed some correction methods to correct the mode-truncated error of the sensitivity of frequency response using the direct differentiation formula) by a convergent power series. Qu [27] presented a correction mode superposition method to solve the problem how many power-series items should be considered to satisfy the necessary accuracy of the sensitivity of frequency response. Model updating based on the frequency response sensitivity of viscously damped systems (to eliminate the output error, often the input force residual with units is considered) has been developed into a mature technology applied successfully to correct the finite element models (see, e.g., Mottershead et al. [5,27], Friswell and Mottershead [28], Friswell and Penny [29]).

The mentioned previous studies only consider viscously damped model. But a physically realistic model of damping mechanism within the scope of linear dynamic analysis (e.g., the damping in composite materials or rubbers) may not be a viscous damping model. Principally speaking, any causal model making the energy dissipation functional nonnegative, may be a candidate for a damping model. Possibly the most general damping model within the scope of linear dynamic analysis is to take into account the hysteretic or frequency-dependent behavior. In this context, often assume that the damping forces depend on the past history of motion via convolution integrals over kernel functions. In 1874, Boltzmann [30] developed a convolution integral (known as *Boltzmann's superposition principle*) of the stress–strain relations of linear isotropic viscoelastic damping materials. Gurtin and Herrera [31] presented the condition that the kernel functions must satisfy to produce dissipative motion. The damping force of such nonviscously damping model can be given by

$$\mathbf{f}_d(t) = \int_0^t \mathbf{g}(t-\tau)\frac{\partial \mathbf{u}(\tau)}{\partial \tau} d\tau \tag{1}$$

where $\mathbf{g}(t)$is the matrix of kernel functions, $\mathbf{u}(t)$ is the displacement vector, $t \in \mathbb{R}^+$ denotes time, τ denotes the integration variable. The kernel functions $\mathbf{g}(t)$ are also used in the literature under many different names, including the constitutive time varying characteristic relaxation functions, after-effect functions, retardation functions, or heredity functions in the context of different subjects. Woodhouse [32] pointed out that the damping model is probably the most generalized damping model within the scope of a linear mechanical analysis. Adhikari [67] gave a significant body of research on such nonvicous damping model, including modeling damping, modal analysis, dynamic response, some insights on system matrix characters and damping identification. In the special case, when $\mathbf{g}(t-\tau) = \mathbf{C}\delta(t-\tau)$ where $\mathbf{C}$ is a viscous damping matrix and $\delta(t-\tau)$ is the Kronecker delta function, Eq. (1) reduces to the case of a viscously damped system. Hence the nonviscous damping model can be considered as a further generalization of the familiar viscous damping. As we can see, the viscous damping model assumes that the instantaneous velocity of the system is the only relevant state variables that determine the damping force, and works reasonably well for lightly damped structures with a small frequency range. It implies that the viscous damping model is a local damping model. The nonviscous damping model considers the entire velocity history of the system via convolution integrals over kernel functions and therefore can be considered as a non-local damping model (it is more likely to have a better match with experimental data), which means the non-local character of time can be considered. It should be mentioned that some authors have considered some damping models with the non-local character of both time and spatial location, for further reading, see Refs. [33–37]. Increasing the use of mechanical structures with