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Controlled synchronization for chaotic systems via limited information with data packet dropout☆

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ABSTRACT

This note addresses a controlled synchronization problem for chaotic systems involving a communication network with data packet dropout. A chaotic master system and its slave system are connected with a controller via a limited channel and data packet dropout is modeled as Bernoulli process. Then, a coder–decoder pair is designed with the controller such that the master system and the slave system are completely synchronized, not necessarily bounded for synchronization. Necessary data capacity of a channel is explicitly stated. Finally, a numerical example for the 3-double-scroll chaotic system is applied to illustrate the effectiveness of the obtained result.

1. Introduction

In the past two decades, the synchronization of chaotic systems has always been a common research topic in non-linear sciences, which has been extensively investigated due to its great potential applications in real world problems, for example, Boccaletti, Kurths, Osipov, Valladares, and Zhou (2002), and the references therein. Many remarkable methods have been proposed on how synchronization can be reached, for instance, Carroll and Pecora (1991), Yassen (2007). Among them, the controlled synchronization is one of the effective methods. This is from a growing interest in applications of control methods in synchronization (Fradkov & Pogromsky, 1998).

Today, control systems involving a communication network have attracted many researchers, and a quantized control is the main concern. There have been many important results reported on this topic. Meanwhile, due to the limited bandwidth and the reliability of a communication network, data packet dropout is inevitable in transmission, and it has been considered recently by Xiong and Lam (2007), Zhang and Yu (2007). It appears that there is less attention paid on quantized synchronization with limited information except for the past few years (Fradkov, Andrievsky, & Evans, 2006, 2008; Fradkov and Andrievsky, 2009 and 2011). However, the quantized synchronization with data packet dropout has seldom been investigated. Since chaos is sensitive on disturbances, chaotic quantized synchronization with data packet dropout is more significant.

Observer-based synchronization with limited information is addressed for continuous-time chaotic systems in Fradkov et al. (2006). In Fradkov et al. (2008), the controlled synchronization of nonlinear Lurie systems with limited information is studied and the results show that the quantized synchronization is bounded and does not tend to zero if the channel required is limited. The complete chaotic quantized synchronization could be reached in case the channel capacity is unlimited. This result is not so satisfactory because a channel capacity is limited in practice. Recently, the passification method is introduced to overcome such a limitation (Fradkov and Andrievsky, 2008, 2009 and 2011). However, no data packet dropout has been considered in these results. To simplify our study, we only consider the state case of quantized synchronization. We will show that the limitation of bounded synchronization could be removed under certain conditions, even it has data packet dropout. This is a main objective of this note. We show that the master system is detectable under a designed coder–decoder pair. Moreover, the channel capacity can be explicitly computed for such synchronization.

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2. Problem formulation

Consider the following general form of chaotic systems:

$$\dot{x}(t) = f(x(t)), \qquad x(0) = x_0, \tag{1}$$

where $x \in R^n$ is the state of the master system. The controlled chaotic system is given by

$$\dot{y}(t) = f(y(t)) + Bu, \qquad y(0) = y_0, \tag{2}$$

where $y \in R^n$ is the state of the slave system, B is a known constant matrix with an appropriate dimension, u is an input. Suppose that the initial state $x_0 \in \Omega$, where Ω is a given bounded set (e.g. the attractor of a chaotic system) and $f(x) \in R^n$ is continuous. The condition on the set Ω is assumed as follows:

$$\|f(x) - f(y)\|_\infty \le L\|x - y\|_\infty.$$

Remark 1. Since Ω is bounded, we assume the Lipschitz condition on Ω, not local as usual, without loss of generality, by following the Lipschitz global optimization method provided by Sergeyev and Kvasov (2010).

In this note, the master and the slave systems are connected via a limited channel. Only a limited number of bits are then transmitted. To design a suitable coder–decoder is the first step. The rectangular coding partition is basically motivated from Savkin and Cheng (2007). Specially, for a given sampling period $T > 0$, the coder signal $h(kT)$ $(k = 1, 2, \ldots)$ is transmitted through a limited channel at discrete time kT. Then, at the remote receiver, a decoder decodes the received code-words and constructs an estimate $\hat{x}$ of the master system's state. If the codeword $h(kT)$ is transmitted successfully, the decoder decodes the received code-words and resets the initial state. On the other hand, if the code-word $h(kT)$ is dropped out, the previous state will be used instead. This generates a discrete variable $\theta_{kT} = 0$ if a measurement is dropped, and $\theta_{kT} = 1$ if a measurement is received. θ_{kT} is an independent and identically distributed Bernoulli process

$$\Pr ob\{\theta_{kT} = 0\} = \delta, \qquad \Pr ob\{\theta_{kT} = 1\} = 1 - \delta, \tag{3}$$

where $0 \le \delta < 1$. We use the following coder–decoder form.

Coder:

$$h(kT) = \mathcal{J}_k\left(x(\cdot)|_0^{kT}\right); \tag{4}$$

Decoder:

$$\hat{x}(t)\Big|_{kT}^{(k+1)T} = \mathcal{L}_k(\theta_T h(T), \theta_{2T} h(2T), \ldots, \theta_{kT} h(kT)), \tag{5}$$

where $\mathcal{J}_k$ and $\mathcal{L}_k$ $(k = 1, 2, \ldots)$ are the undetermined coder and decoder functions. Meanwhile, it is assumed that the information on data is lost or not is known to the decoder.

3. Main results

Definition 1. The master system (1) is said to be detectable via a limited channel, if there exists a coder–decoder (4) and (5) such that

$$\lim_{t\to\infty} E\{\|x(t) - \hat{x}(t)\|_\infty\} = 0,$$

where E is a mathematical expectation.

Definition 2. The quantized synchronization between (1) and (2) is reached via a limited channel if there exists a coder–decoder pair (4) and (5) such that

$$\lim_{t\to\infty} E\{\|x(t) - y(t)\|_\infty\} = 0.$$

Suppose that the number N taken by the coder satisfies $N = q^n$, where q is a positive integer. For any given constant $a > 0$,

$$B(0, a) = \left\{x \in R^n \,\Big|\, ||x||_\infty \le a\right\}$$

is partitioned into q^n equal super-cube boxes

$$I_{i_1}^1(a) \times I_{i_2}^2(a) \times \cdots \times I_{i_n}^n(a),$$

where $i_j \in \{1, 2, \ldots, q\}$ $(j = 1, 2, \ldots, n)$. For each $i \in \{1, 2, \ldots, n\}$, the corresponding column state x_i of x is located in one of q intervals as follows:

$$I_1^i(a) := \left\{x_i : -a \le x_i < -a + \frac{2a}{q}\right\},$$

$$I_2^i(a) := \left\{x_i : -a + \frac{2a}{q} \le x_i < -a + \frac{4a}{q}\right\}, \ldots$$

$$I_q^i(a) := \left\{x_i : a - \frac{2a}{q} \le x_i \le a\right\}.$$

Then, for any given $x \in B(0, a)$, there exists a group of integers $i_1, i_2, \ldots, i_n \in \{1, 2, \ldots, q\}$ such that

$$x \in I_{i_1}^1(a) \times I_{i_2}^2(a) \times \cdots \times I_{i_n}^n(a) \subset B(0, a). \tag{6}$$

Define the center of $I_{i_1}^1(a) \times I_{i_2}^2(a) \times \cdots \times I_{i_n}^n(a)$, containing the state x as follows.

$$C_a(i_1, i_2, \ldots, i_n) = \left[-a + \frac{2i_1 - 1}{q}a, -a + \frac{2i_2 - 1}{q}a, \ldots, -a + \frac{2i_n - 1}{q}a\right]^T.$$

Denote

$$M_0 = \sup_{x_0\in\Omega} \|x_0\|_\infty, \qquad a(0) = M_0, \qquad a(1) = \frac{e^{LT}}{q}M_0$$

and

$$a(k+1) = \left(1 - \left(1 - \frac{1}{q}\right)\theta_{kT}\right)e^{LT}a(k), \quad k = 1, 2, \ldots. \tag{7}$$

Remark 2. Assume that the first coder signal is transmitted successfully.

Now we are in a position to design the coder–decoder pair, which is given by

Coder: $h(kT) = \{i_1, i_2, \ldots, i_n\}$ for

$$x(kT) - \hat{x}^-(kT) \in I_{i_1}^1(a(k)) \times I_{i_2}^2(a(k)) \times \cdots \times I_{i_n}^n(a(k)), \tag{8}$$

where "$^-$" refers to the limit from below.

Decoder:

$$\begin{cases}\dot{\hat{x}}(t) = f(\hat{x}(t)), \quad t \in [kT, (k+1)T), \qquad \hat{x}^-(0) = 0, \\ \hat{x}(kT) = \hat{x}^-(kT) + \theta_{kT} C_{a(k)}(i_1, i_2, \ldots, i_n).\end{cases} \tag{9}$$

Theorem 1. *For the given data packet dropout rate δ, if $\delta e^{LT} < 1$, then the master system (1) is detectable under the given coder–decoder pair.*

Proof. Since $\delta e^{LT} < 1$, then, there exists an integer $q > 0$ such that

$$r = \left(\delta + \frac{1-\delta}{q}\right)e^{LT} < 1. \tag{10}$$

We first show that the decoding condition satisfies

$$x(kT) - \hat{x}^-(kT) \in B(0, a(k)) \tag{11}$$