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Synchronization and power sharing for droop-controlled inverters in islanded microgrids[☆]

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ABSTRACT

Motivated by the recent and growing interest in smart grid technology, we study the operation of DC/AC inverters in an inductive microgrid. We show that a network of loads and DC/AC inverters equipped with power-frequency droop controllers can be cast as a Kuramoto model of phase-coupled oscillators. This novel description, together with results from the theory of coupled oscillators, allows us to characterize the behavior of the network of inverters and loads. Specifically, we provide a necessary and sufficient condition for the existence of a synchronized solution that is unique and locally exponentially stable. We present a selection of controller gains leading to a desirable sharing of power among the inverters, and specify the set of loads which can be serviced without violating given actuation constraints. Moreover, we propose a distributed integral controller based on averaging algorithms, which dynamically regulates the system frequency in the presence of a time-varying load. Remarkably, this distributed-averaging integral controller has the additional property that it preserves the power sharing properties of the primary droop controller. Our results hold for any acyclic network topology, and hold without assumptions on identical line admittances or voltage magnitudes.

1. Introduction

A *microgrid* is a low-voltage electrical network, heterogeneously composed of distributed generation, storage, load, and managed autonomously from the larger primary network. Microgrids are able to connect to the wide area electric power system (WAEPS) through a Point of Common Coupling (PCC), but are also able to "island" themselves and operate independently. Energy generation within a microgrid can be highly heterogeneous, and many typical sources generate either variable frequency AC power (wind) or DC power (solar). To interface with a synchronous AC microgrid, these sources are connected via power electronic devices called DC/AC (or AC/AC) *power converters*, or simply *inverters*. In islanded operation, inverters are operated as *voltage sourced inverters* (VSIs), which act much like ideal voltage sources. It is through these VSIs that control actions must be taken to ensure synchronization, security, power balance and load sharing in the network.

Literature review: A key topic of interest within the microgrid community is that of accurately sharing both active and reactive power among a bank of inverters operated *in parallel*. Such a network is depicted in Fig. 1, in which each inverter transmits power directly to a common load. Although several control architectures have been proposed to solve this problem, the so-called "droop" controllers have attracted the most attention, as they are ostensibly decentralized. The original reference for this methodology is Chandorkar, Divan, and Adapa (1993), where Chandorkar et al. introduce what we will refer to as the *conventional droop controller*. For inductive lines, the conventional droop controller attempts to emulate the behavior of a classical synchronous generator by imposing an inverse relation at each inverter between frequency and active power injection (Kundur, 1994). Under other network conditions, the controller takes different forms (Guerrero, GarciadeVicuna, Matas, Castilla, & Miret, 2005; Yao, Chen, Matas, Guerrero, & Qian, 2011; Zhong & Hornik, 2013). Some representative references for the basic methodology are Barsali, Ceraolo, Pelacchi, and Poli (2002), Guerrero, Vasquez, Matas, Castilla, and de Vicuna (2009), Li and Kao (2009), Lopes, Moreira, and Madureira (2006), Majumder, Ghosh, Ledwich, and Zare (2008), and Tuladhar, Jin, Unger, and Mauch (1997). Small-signal stability analyses for two inverters operating in parallel are presented under various assumptions in Coelho, Cortizo, and Garcia (2002), Dai, Marwali, Jung, and Keyhani (2004), Marwali,

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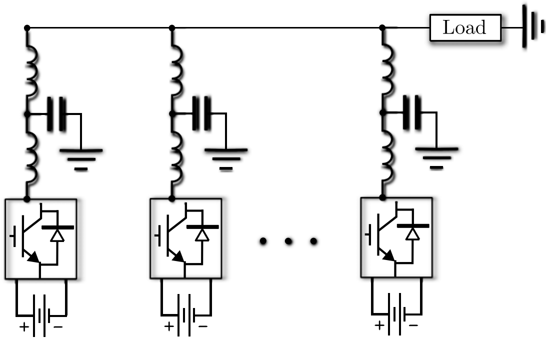

Fig. 1. Schematic of inverters operating in parallel.

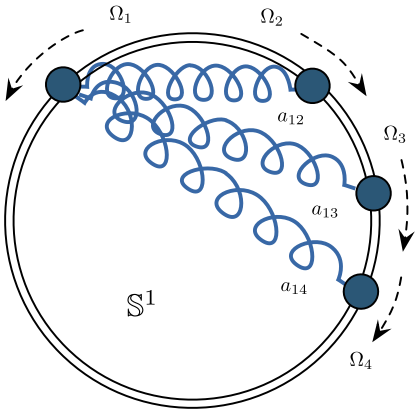

Fig. 2. Mechanical analog of a Kuramoto oscillator network. The particles have no inertia and do not collide with one another.

Jung, and Keyhani (2007), Mohamed and El-Saadany (2008) and the references therein. The recent work (Zhong, 2013) highlights some drawbacks of the conventional droop controller. Distributed controllers based on tools from synchronous generator theory and multi-agent systems have also been proposed for synchronization and power sharing. See Olfati-Saber, Fax, and Murray (2007) and Ren, Beard, and Atkins (2007) for a broad overview, and Bolognani and Zampieri (in press), Tôrres, Hespanha, and Moehlis (2012), Xin, Qu, Seuss, and Maknouninejad (2011) and Zhong and Hornik (2013) for various works.

Another set of literature relevant to our investigation is that pertaining to synchronization of phase-coupled oscillators, in particular the classic and celebrated *Kuramoto model*. A generalization of this model considers $n \geq 2$ coupled oscillators, each represented by a phase $\theta_i \in \mathbb{S}^1$ (the unit circle) and a natural frequency $\Omega_i \in \mathbb{R}$. The system of coupled oscillators obeys the dynamics

$$D_i\dot{\theta}_i = \Omega_i - \sum_{j=1}^{n} a_{ij}\sin(\theta_i - \theta_j), \quad i \in \{1, \ldots, n\}, \tag{1}$$

where $a_{ij} \geq 0$ is the coupling strength between the oscillators i and j and D_i is the time constant of the ith oscillator. Fig. 2 shows a mechanical analog of (1), in which the oscillators can be visualized as a group of n kinematic particles, constrained to rotate around the unit circle. The particles rotate with preferred directions and speeds specified by the natural frequencies Ω_i, and are connected together by elastic springs of stiffness a_{ij}. The rich dynamic behavior of the system (1) arises from the competition between the tendency of each oscillator to align with its natural frequency Ω_i, and the synchronization enforcing coupling $a_{ij}\sin(\theta_i - \theta_j)$ with its neighbors. We refer to the recent surveys (Arenas, Díaz-Guilera, Kurths, Moreno, & Zhou, 2008; Dörfler & Bullo, 2011; Strogatz, 2000) for applications and theoretic results.

The frequency-droop controller: The *frequency-droop controller* constitutes one half of the conventional droop control strategy, with the other half concerning the interplay between reactive power and voltage magnitudes (see Sections 6–7). For inductive lines, the controller balances the active power demand in the network by instantaneously changing the frequency ω_i of the voltage signal at the ith inverter according to

$$\omega_i = \omega^* - n_i(P_{e,i} - P_i^*), \tag{2}$$

where ω^* is a rated frequency, $P_{e,i}$ is the active electrical power injection at inverter i, and P_i^* is the inverters nominal active power injection. The controller gain $n_i > 0$ is referred to as the *droop coefficient*.

Limitations of the literature: Despite forming the foundation for the operation of parallel VSIs, the frequency-droop control law (2) has never been subject to a nonlinear analysis (Zhong, 2013). No conditions have been presented under which the controller (2) leads the network to a synchronous steady state, nor have any statements been made about the convergence rate to such a steady state should one exist. Stability results that are presented rely on linearization for the special case of two inverters, and sometimes come packaged with extraneous assumptions (Guerrero et al., 2009; Majumder et al., 2008). No guarantees are given in terms of performance. Schemes for power sharing based on ideas from multi-agent systems often deal directly with coordinating the real and reactive power injections of the distributed generators, and assume implicitly that a low level controller is bridging the gap between the true network physics and the desired power injections. Moreover, conventional schemes for frequency restoration typically rely on a combination of local integral action and separation of time scales, and are generally unable to maintain an appropriate sharing of power among the inverters (see Sections 4–5).

Contributions: The contributions of this paper are four-fold. First, we begin with our key observation that the equations governing a microgrid under the frequency-droop controller can be equivalently cast as a generalized Kuramoto model of the form (1). We present a necessary and sufficient condition for the existence of a locally exponentially stable and unique synchronized solution of the closed-loop, and provide a lower bound on the exponential convergence rate to the unique synchronized solution. We also state a robustified version of our stability condition which relaxes the assumption of fixed voltage magnitudes and admittances. Second, we show rigorously — and without assumptions on large output impedances or identical voltage magnitudes — that if the droop coefficients are selected proportionally, then power is shared among the units proportionally. We provide explicit bounds on the set of serviceable loads. Third, we propose a distributed "secondary" integral controller for frequency stabilization. Through the use of a distributed-averaging algorithm, the proposed controller dynamically regulates the network frequency to a nominal value, while preserving the proportional power sharing properties of the frequency-droop controller. We show that this controller is locally stabilizing, without relying on the classic assumption of a time-scale separation between the primary (droop) and secondary (integral) control loops. Fourth and finally, all results presented extend past the classic case of a parallel topology of inverters and hold for generic acyclic interconnections of inverters and loads.

Paper organization: The remainder of this section introduces some notation and reviews some fundamental material from algebraic graph theory, power systems and coupled oscillator theory. In Section 2 we motivate the mathematical models used throughout the rest of the work. In Section 3 we perform a nonlinear stability analysis of the frequency-droop controller. Section 4 details results on