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Coverage control in constant flow environments based on a mixed energy–time metric[☆]

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ABSTRACT

In this paper, we study a multi-vehicle coverage control problem in constant flow environments while taking into account both energy consumption and traveling time. More specifically, the metric (called the mixed energy–time metric) is a weighted sum of the energy consumption and the traveling time for a vehicle to travel from one point to another in constant flows when using the minimum energy control, and the objective is to find vehicle locations that can minimize the expected mixed energy–time required for the set of vehicles to cover a region. We propose a gradient based control law which is calculated based on refined approximated Voronoi cells (induced by the mixed energy–time metric) and of which the convergence is proved via Hybrid Systems Theory. Simulations show that the refined gradient based control can achieve similar performance as the exact gradient based control.

1. Introduction

Large groups of mobile vehicles equipped with sensors (such as the ones reported in D'Este, Barnes, Sharman, & McCulloch, 2012; Floating sensor network project, 0000) are currently being developed to facilitate surveillance and actuation in remote environments. In flow environments, monitoring tasks include the evaluation of algae blooms (Bertozzi, Kemp, & Marthaler, 2005), debris (Lee, Cho, King, Fang, & Lee, 2009), and water levels at flood season (Zhang, Tao, & Cao, 2010). These monitoring tasks require mobile vehicles to move to a certain point of interest and take actions. For nonurgent tasks like the monitoring of algae blooms in rivers, the top priority is to preserve energy consumption while vehicles reach the point of interest to take measurements of certain chemicals. However, in search and rescue missions, or in rapidly changing environments, vehicles need to reach the point of interest within the shortest time period. For other tasks in between, there should be a compromise between minimizing traveling time and minimizing energy consumption. Motivated by this, here we propose a mixed energy–time metric to capture this compromise, and investigate the optimal locations of mobile vehicles to cover a certain region.

Deployment of a group of mobile vehicles to collectively cover a certain region has been studied extensively (Anderson, Bakolas, Milutinović, & Tsiotras, 2012; Barrett, 2007; Baumgartner, Ferrari, & Rao, 2009; Cortés, Martinez, Karatas, & Bullo, 2004; Enright, Savla, & Frazzoli, 2008; Hokayem, Stipanović, & Spong, 2007; Kwok & Martinez, 2010; Luna, Fierro, Abdallah, & Wood, 2010; Mahboubi, Sharifi, Aghdam, & Zhang, 2012; Stanković, Dürr, & Johansson, 2011) (for a more comprehensive treatment, refer to Chapter 5 in Bullo, Cortes, & Martinez, 2009). The objective is usually to maximize/minimize a function related to the sensing performance (e.g., the work in Cortés et al., 2004; Hokayem et al., 2007; Luna et al., 2010; Mahboubi et al., 2012; Stanković et al., 2011) or the traveling time (e.g., the work in Anderson et al., 2012; Enright et al., 2008; Kwok & Martinez, 2010). The underlying vehicle models can be either holonomic (e.g., Cortés et al., 2004; Kwok & Martinez, 2010; Luna et al., 2010) or nonholonomic (e.g., Enright et al., 2008). There is limited work on coverage control in river environments (one example is the work in Kwok & Martinez, 2010). In Kwok and Martinez (2010), mobile vehicles cannot run against flow and the objective is to deploy them to maximize the total area reachable in a fixed amount of time. For nonurgent tasks, the energy consumption of mobile vehicles powered by batteries can be more critical compared with the traveling time due to

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limited battery capacity. In Ru and Martinez (2011), we study the minimum energy control for holonomic vehicles in constant flow environments and propose a minimum energy metric to assign regions of the environment to them (for details, refer to Proposition 2).

In this paper, we study a coverage control problem in river environments while taking into account both traveling time and energy consumption. This is motivated by the facts that mobile vehicles are usually powered by batteries of limited capacity and for certain relatively urgent tasks the traveling time also needs to be taken into account. Therefore, we introduce a mixed energy–time metric based on the minimum energy control for mobile vehicles in constant flows, to capture a compromise between traveling time and energy consumption. Our objective is to drive a group of vehicles to a vehicle location configuration that locally minimizes this metric. Since the mixed metric induces Voronoi partitions that might not be convex and are difficult to compute exactly, we introduce lower and upper approximations that can be refined to be arbitrarily close to the exact partition. We propose a refined gradient based control law and prove its convergence by formulating vehicle evolutions as a hybrid system and utilizing the Hybrid Invariance Principle (Sanfelice, Goebel, & Teel, 2005). In our algorithm, region refinement is produced when a condition that guarantees objective minimization is violated, similarly in spirit to the self-triggered strategy of Nowzari and Cortés (2011) for the sporadic update of vehicles' locations. Thus, our approach could be combined with Nowzari and Cortés (2011) and provides a general methodology to deal with general metrics for which Voronoi regions are hard to compute but which can be approximated with an arbitrary precision at the expense of higher computational costs. Simulations show that the refined gradient based control can achieve similar performance as the exact gradient based control. The contributions of this work include (i) the introduction of a mixed energy–time metric for vehicles in constant flow environments, (ii) the proposed (refined) lower and upper approximations of mixed Voronoi cells, (iii) the introduction of a general methodology (i.e., the result in Theorem 5) to deal with hard-to-compute Voronoi regions in coverage problems, and (iv) the proof of the convergence for refined gradient-based control.

2. Problem formulation

The studied flow environment is described by a bounded two-dimensional region $D = \left\{(x\, y)^T \in \mathbb{R}^2 \mid 0 \le x \le L,\ |y| \le \frac{W}{2}\right\}$, where $L > 0$ (or $W > 0$) is the length (or width) of the region. The velocity field is a mapping $v : D \to \mathbb{R}^2$ which maps $(x\, y)^T$ to $(B\, 0)^T$ with $B > 0$, i.e., the flow is only in the x direction.

A vehicle runs at speed $u = (u_x\, u_y)^T$, and then its dynamics can be described by

$$\frac{dx}{dt} = u_x + B, \qquad \frac{dy}{dt} = u_y. \tag{1}$$

We assume that vehicles can run against the flow. To quantify the minimum energy required for a vehicle to move from one point to another in the region D among all possible controls, we recall a (pseudo)-metric as introduced in Ru and Martinez (2011).

Definition 1. Given two points p^1 and p^2 in the flow environment D, the energy metric $J(p^1, p^2)$ is defined as $J(p^1, p^2) = \min \int_0^{t_f} u^T u dt$, where t_f is free, u satisfies Eq. (1), and $x(0) = x_{p^1}$ (i.e., the x coordinate of p^1), $y(0) = y_{p^1}$ (i.e., the y coordinate of p^1), $x(t_f) = x_{p^2}$, $y(t_f) = y_{p^2}$.

The explicit expression for the energy metric is given below, and the minimum energy control results in a straight line trajectory from p^1 to p^2 with the traveling time $t_f = \frac{d_{p^1p^2}}{B}$.

Proposition 2 (Ru & Martinez, 2011). *Given p^1 and p^2 in D with the velocity field $v = (B\, 0)^T$, the minimum energy is*

$$J(p^1, p^2) = 2B(d_{p^1p^2} + x_{p^1} - x_{p^2}),$$

and the optimal control is $u(t) = -\frac{1}{2}(C_1\, C_2)^T$ for $t \in [0, t_f]$, where $C_1 = 2B\left(1 + \frac{x_{p^1} - x_{p^2}}{d_{p^1p^2}}\right)$, $C_2 = \frac{2B(y_{p^1} - y_{p^2})}{d_{p^1p^2}}$, and $t_f = \frac{d_{p^1p^2}}{B}$. □

Given a set of n vehicles $\{1, 2, \dots, n\}$ with locations $P = \{p^1, p^2, \dots, p^n\}$ and a task at $q \in D$, we are interested in assigning a vehicle from P to serve the task using *the minimum energy control*. We do so using a multi-center function (Bullo et al., 2009). To capture the importance of the location q, we consider a continuous density function $\phi : D \to \mathbb{R}_{\ge 0}$. The larger the value $\phi(q)$, the more important the location q is. Analogously to Bullo et al. (2009), one can use

$$\mathcal{H}_1(P) = \int_D \min_{p^i \in P} J(p^i, q)\phi(q) dq$$

to determine a locally optimal sensor configuration and region partition. At a local minimum, a task at q is assigned to p^i if and only if it can be reached with smaller energy from p^i than from any other p^j using the minimum-energy control of Proposition 2. Alternatively, let $t(p^i, q)$ be the traveling time from p^i to q along a straight line with given velocity upper limit w (i.e., the *minimum-time control* with velocity w). Similarly to what has been done in multi-objective optimization, one could consider the mixed objective $\int_D \min_{p^i \in P}(\beta J(p^i, q) + (1 - \beta)t(p^i, q))\phi(q)dq$ to balance between energy consumption and traveling time to serve q, where $\beta \in [0, 1]$. However, it is not clear how p^i should be controlled to reach an assigned q under this cost as the underlying control strategies (minimum energy/minimum time) are different — or how to find u with the mixed cost

$$\beta J(p^i, q) + (1 - \beta)t(p^i, q). \tag{2}$$

Using exclusively the minimum-energy control, we can factor in a time consideration in $\mathcal{H}_1$ as follows. For $\beta \in [0, 1]$, define:

$$J_{\text{mix}}(p^1, p^2) = \beta J(p^1, p^2) + (1 - \beta)t_f. \tag{3}$$

Then $\min_{p^i \in P} J_{\text{mix}}(p^i, q)$ is the minimum mixed energy–time required for the set of vehicles to serve the task at q using the minimum-energy control. The expected minimum mixed energy–time for P to cover D is then given as:

$$\mathcal{H}(P) = \int_D \left(\min_{p^i \in P} J_{\text{mix}}(p^i, q)\right) \phi(q) dq. \tag{4}$$

Observe that, when $\beta = 0$, a locally optimal solution to $\mathcal{H}$ assigns q to p^i if and only if it can be reached with less time from p^i than from any other p^j but using the minimum-energy control. Other mixed metrics can be defined by choosing alternative motion control strategies. For example, using the minimum-time control with velocity w to go from p^i to q induces an energy expenditure of $\tilde{J}(p^i, q)$. Then $\tilde{J}_{\text{mix}}(p^i, q) = \beta \tilde{J}(p^i, q) + (1-\beta)t(p^i, q)$ would be the corresponding mixed pseudo-distance factoring in an energy consideration in the assignment. The difference between J_{mix} and $\tilde{J}_{\text{mix}}$ lies in the "priority" which is given to energy consumption versus time expenditure.

Remark 3. A similar cost function is considered in Baumgartner et al. (2009), where a linear combination of track coverage and energy consumption is used as an objective to simultaneously maximize the quality of service and minimize the power consumption. In general, since the traveling time and the energy consumption are correlated, multi-objective optimization is necessary to capture the Pareto frontier; one approach is to use genetic algorithms as in Barrett (2007).