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Robust decentralized dynamic optimization at presence of malfunctioning agents[☆]

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ABSTRACT

This paper considers the problem of tracking a network-wide solution that dynamically minimizes the summation of time-varying local cost functions of network agents, when some of the agents are malfunctioning. The malfunctioning agents broadcast faulty values to their neighbors, and lead the optimization process to a wrong direction. To mitigate the influence of the malfunctioning agents, we propose a total variation (TV) norm regularized formulation that drives the local variables of the regular agents to be close, while allows them to be different with the faulty values broadcast by the malfunctioning agents. We give a sufficient condition under which consensus of the regular agents is guaranteed, and bound the gap between the consensual solution and the optimal solution we pursue as if the malfunctioning agents do not exist. A fully decentralized subgradient algorithm is proposed to solve the TV norm regularized problem in a dynamic manner. At every time, every regular agent only needs one subgradient evaluation of its current local cost function, in addition to combining messages received from neighboring regular and malfunctioning agents. The tracking error is proved to be bounded, given that variation of the optimal solution is bounded. Numerical experiments demonstrate the robust tracking performance of the proposed algorithm at presence of the malfunctioning agents.

1. Introduction

Consider an undirected network consisting of n agents, which at time k try to cooperatively solve a decentralized dynamic optimization problem

$$\min_{\tilde{x}^k} \sum_{i=1}^{n} f_i^k(\tilde{x}^k). \tag{1}$$

Here $f_i^k : \mathbb{R}^p \to \mathbb{R}$ is a convex and differentiable local cost function only available to agent i at time k and $\tilde{x}^k \in \mathbb{R}^p$ is the common optimization variable to all agents. At time k, every agent is allowed to exchange its current local iterate with network neighbors, followed by local computation so as to track the dynamic optimal solution. The purpose of this paper is to develop a robust decentralized dynamic optimization algorithm that solves (1) at presence of malfunctioning agents. By malfunctioning agents, we mean those who, instead of transmitting local iterates to neighbors, send wrong values (for example, faulty constants or random variables) due to failures of communication or computation units.

Decentralized dynamic optimization problems in the form of (1) are popular in multi-agent networks with time-varying tasks [2–5]. Examples include adaptive filtering and estimation in a wireless sensor network [6–8], target tracking using a group of robots [9–11], dynamic resource allocation over a communication network [12–14], voltage control of a power network [15,16], to name a few. Existing algorithms to solve (1) are (sub)gradient methods [8,15], mirror descent method [5], alternating direction method of multipliers [2,14], as well as gradient, Newton, and interior point methods based on the prediction-correction scheme [3,4].

Nevertheless, most of the existing works assume that the agents faithfully follow prescribed optimization protocols: accessing dynamic local cost functions, exchanging local iterates, and performing local computations. This assumption does not always hold true since some of the agents might be malfunctioning in practice – some may send malicious information to their neighbors so as to deliberately guide the optimization process to a wrong direction that they expect to reach, whilst some may send faulty values to

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their neighbors, not deliberately but due to failures of communication or computation units. This paper focuses on mitigating the impact of malfunctioning agents in decentralized dynamic optimization.

The impact of malfunctioning agents has been analyzed in the context of average consensus over a social network [17–19]. It is shown that the malfunctioning agents shall bias the network opinions from the consensual state of the regular agents [17], and the locations of the malfunctioning agents are critical to evolution of the network opinions [18]. Decentralized detection and localization methods are proposed in [19] to identify the malfunctioning agents. To the best of our knowledge, there is no existing work that considers the influence of the malfunctioning agents on the tracking performance of decentralized dynamic optimization.

Our work is tightly related to [20], whose goal is decentralized *static* optimization at presence of the malfunctioning agents. Different from the *dynamic* case studied in this paper, Ben-Ameur et al. [20] assumes that the local cost functions f_i^k are invariant across time k. To handle the faulty values broadcast by the malfunctioning agents, the total variation (TV) norm of the vector that stacks all the local variables is penalized. Through minimizing the summation of the local cost functions and the TV norm, most local variables (from the regular agents) are able to reach consensus and those outliers (from the malfunctioning agents) are tolerated. A subgradient method is proposed to solve this robust decentralized *static* optimization problem. Our work also adopts the TV norm penalty to handle the malfunctioning agents and a subgradient algorithm as the optimization tool, but extends their applications to the *dynamic* regime. We give a sufficient condition under which consensus of the regular agents is guaranteed, and also give an upper bound on the tracking error of the regular agents. These results are different to those developed for the *static* case in [20] due to the *dynamic* environment, and provide theoretical guarantees to the tracking performance of the subgradient method at presence of the malfunctioning agents.

Another related work is [21], which considers decentralized *stochastic* optimization. Instead of tracking a dynamic optimal solution, Koppel et al. [21] minimizes the summation of the local cost functions f_i^k across all nodes i and all times k. Therefore, the local iterates are expected to reach a steady-state consensual solution, given that the stochastic noise of the local cost functions is bounded. To allow for data heterogeneity across the network, Koppel et al. [21] introduces proximity constraints such that neighboring local variables are close enough, but not necessarily consensual. Though not explicitly claimed in [21], this approach is also able to alleviate the influence of the malfunctioning agents. A saddle point method is proposed to solve this constrained stochastic optimization problem. Our work is different from [21] in terms of problem setting (dynamic versus stochastic), mathematical formulation (TV norm penalty versus proximity constraints), and algorithm design (subgradient versus saddle point).

The main contributions of this paper are as follows.

1. We formulate a TV norm regularized problem, which is robust to presence of the malfunctioning agents (Section 2). We give a sufficient condition under which consensus of the regular agents is guaranteed, and bound the gap between the consensual solution and the optimal solution we pursue as if the malfunctioning agents do not exist (Section 3.2).
2. We propose a fully decentralized subgradient algorithm to solve the TV norm regularized problem in a dynamic manner. At every time, every regular agent only needs one subgradient evaluation, in addition to combining messages from neighboring regular and malfunctioning agents (Section 2). We prove that the tracking error is bounded, given that the variation of the optimal solution is bounded (Section 3.3).
3. We provide extensive numerical experiments, demonstrating the robust tracking performance of the proposed algorithm at presence of the malfunctioning agents (Section 4).

2. Problem formulation and algorithm design

Let us consider a connected undirected network of n agents $\mathcal{V} = \{1, \cdots, n\}$ with $n = |\mathcal{V}|$, and a set of edges $\mathcal{A}$. If an edge $(i, j) \in \mathcal{A}$, then agents i and j are neighbors, and can communicate with each other. We denote the set of agent i's neighbors as $\mathcal{N}_i$. The agents aim at solving the decentralized dynamic optimization problem in the form of (1). We assume that the network is synchronized, and at time k every agent i strictly conforms to the following protocol:

Step 1. Accessing local cost function f_i^k.

Step 2. Computing local iterate $x_i^k \in \mathbb{R}^p$.

Step 3. Broadcasting local iterate x_i^k to neighbors $j \in \mathcal{N}_i$.

However, some of the agents in the network are malfunctioning, meaning that they broadcast faulty values other than local iterates. To be specific, denote $\mathcal{M}$ as the set of malfunctioning agents and $\mathcal{R} := \mathcal{V} \backslash \mathcal{M}$ as the set of regular agents. Define $r := |\mathcal{R}|$ and $m := |\mathcal{M}|$. The subset of edges connecting the regular agents in $\mathcal{V}$ is denoted by $\mathcal{E} \subseteq \mathcal{A}$. At time k, malfunctioning agent $i \in \mathcal{M}$ broadcasts a variable $z_i^k \in \mathbb{R}^p$, instead of x_i^k, to its neighbors $j \in \mathcal{N}_i$. The faulty value may come from deliberate malicious attack, failure of the computation unit, or breakdown of the communication unit. Different from [17–20] that assume the faulty values are constant across time k, we also allow that they are time-varying (for example, random variables or values generated from certain functions of time). Although identifying the malfunctioning agents is possible in decentralized *static* optimization [19], their detection and localization are much more challenging for the *dynamic* task, especially when the faulty values are time-varying.

Observe that at presence of the malfunctioning agents, it is meaningless to solve (1), which minimizes the summation of all agents' local cost functions. For example, in multi-robot tracking, when several robots are malfunctioning, taking their information into consideration shall bias the tracking result. Therefore, at time k, our goal is no longer solving (1) but finding the dynamic optimal solution that minimizes the summation of the regular agents' local cost functions

$$\tilde{x}^{k*} := \arg\min_{\tilde{x}^k} \sum_{i\in\mathcal{R}} f_i^k(\tilde{x}^k). \tag{2}$$

Directly solving (2) is intractable because the identities of malfunctioning agents are not available in advance. To address this issue, we introduce a TV norm penalty on the transmitted values, which include the local iterates of the regular agents and the faulty values from the malfunctioning agents. For agent i, define $\mathcal{R}_i$ as the set of its regular neighbors and $\mathcal{M}_i := \mathcal{N}_i \backslash \mathcal{R}_i$ as the set of its malfunctioning neighbors. At time k, we expect to approximately solve

$$x^{k*} := [x_i^{k*}] = \arg\min_{x^k := [x_i^k]} \sum_{i\in\mathcal{R}} f_i^k(x_i^k) + \lambda \sum_{i\in\mathcal{R}} \left(\frac{1}{2} \sum_{j\in\mathcal{R}_i} \|x_i^k - x_j^k\|_1 + \sum_{j\in\mathcal{M}_i} \|x_i^k - z_j^k\|_1 \right), \tag{3}$$

where $x^k := [x_i^k] \in \mathbb{R}^{rp}$ is a vector that stacks all the local variables x_i^k of regular agents, $x^{k*} := [x_i^{k*}] \in \mathbb{R}^{rp}$ is the optimal solution of (3), and λ is a positive constant penalty factor. The second term in the cost function of (3) is the TV norm penalty on the transmitted values, whose minimization forces every x_i^k to be close to most of the received values on agent i, but allows it to be different to those received outliers [20]. Therefore, when the malfunctioning agents are sparse within the network, the TV norm penalty helps mitigate their negative influence. For the applications of TV norm in identifying sparse outliers, readers are referred to [22,23].