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Subspace-based fault detection robust to changes in the noise covariances☆

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ABSTRACT

The detection of changes in the eigenstructure of a linear time invariant system by means of a subspace-based residual function has been proposed previously. While enjoying some success in its applicability in particular in the context of vibration monitoring, the robustness of this framework against changes in the noise properties has not been properly addressed yet. In this paper, a new robust residual is proposed and the robustness of its statistics against changes in the noise covariances is shown. The complete theory for hypothesis testing for fault detection is derived and a numerical illustration is provided.

1. Introduction

The family of subspace identification algorithms has been investigated and has expanded rapidly since its foundations in the 1980's (Benveniste & Fuchs, 1985; Larimore, 1983; van Overschee & De Moor, 1996; Verhaegen, 1993; Viberg, 1995; Viberg, Wahlberg, & Ottersten, 1997). Fault detection and isolation (FDI) is another topic closely related to system identification (Delyon & Benveniste, 1997). In many applications, the FDI problem is a crucial issue which has been theoretically and experimentally investigated with different types of approaches, as can be seen from the survey papers (Basseville, 1998, 2009; Frank, 1990; Gertler, 1988; Isermann, 1984; Willsky, 1976), the books (Basseville & Nikiforov, 1993; Patton, Frank, & Clarke, 1989), etc.

In several fault detection problems, the detection of changes in the eigenstructure of a linear time invariant system is the main subject of interest, as for health monitoring of mechanical systems and vibrating structures (Doebling, Farrar, & Prime, 1998; Farrar, Doebling, & Nix, 2001), where subspace methods have exhibited good capabilities in eigenstructure identification (Basseville et al., 2001). Fault detection for vibrating structures has also concurrently been investigated over the years using very different methods (Isermann, 2006). A particular approach based on subspace identification methods has been developed in Basseville, Abdelghani, and Benveniste (2000), which belongs to the family of model-based approaches that have been considered for a long time (Isermann, 2005; Isermann & Balle, 1997). The approach consists in the detection of small deviations from a reference system based on the so-called local approach (Benveniste, Basseville, & Moustakides, 1987; Le Cam, 1986), where the fault detection problem for a parameterized stochastic process is transformed into monitoring the mean of a Gaussian residual vector. The subspace-based residual uses the left null space of a nominal observability matrix of the system in a reference state, which is the same as a corresponding Hankel matrix built from the reference output data (Viberg, 1995). In a possibly faulty state the residual denotes whether the new Hankel matrix is still well described by the null space of the reference state. A comparison through a χ^2 test is possible because

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the covariance of the residual is assumed to be the same under both hypotheses in Basseville et al. (2000).

There are a number of convergence studies on subspace identification methods in a stationary context in the literature, for example in Bauer (2005), Bauer, Deistler, and Scherrer (1999), Bauer and Jansson (2000), Chiuso and Picci (2004), Chiuso and Picci (2005) and Deistler, Peternell, and Scherrer (1995). Therefore, the behavior and robustness of subspace methods in the context of identification is well understood. However, the problem of changes in the noise statistics for the output-only fault detection framework developed in Basseville et al. (2000) has not been addressed, assuming that the non-stationary consistency property of subspace methods (Benveniste & Fuchs, 1985) is inherited. For a stationary system, the assumption about constant noise covariance during several measurements cannot be sustained in a real life context and is revisited in this paper. While changes in the noise characteristics do not affect the system eigenstructure, they affect the statistics of the considered residual function. Unlike in Gustafsson (1996), where the problem of detecting changes in the pole and the zero parts of a transfer function is addressed, our concern is the output-only detection of changes in the eigenstructure, while asking for robustness with respect to changes in the zeros of the system. Note that the only attempt to generalize the FDI approach of Basseville et al. (2000) to input/output subspace methods is the recent work of Esna Ashari and Mevel (2012).

The paper is organized as follows. In Section 2, the state of art for subspace-based fault detection using the local approach is recalled. The weakness of this approach in the context of changing process noise covariance is analyzed in Section 3. In Section 4, a new subspace residual based on the left factor of the singular value decomposition of the Hankel matrix is proposed and its robustness to changes in the process noise covariance is proven. In Section 5, the properties of the corresponding test are investigated, as well as necessary rank properties. Finally, the efficiency of the new residual with respect to previous approaches is illustrated on a numerical example from vibration monitoring in Section 6.

2. Stochastic subspace-based fault detection

In this section, the residual function of Basseville et al. (2000) is recalled that is based on well established subspace foundations (Viberg et al., 1997).

2.1. Models and parameters

Consider the discrete time state–space model

$$\begin{cases} x_{k+1} = Ax_k + v_k \\ y_k = Cx_k + w_k \end{cases} \tag{1}$$

with the states $x_k \in \mathbb{R}^n$, the outputs $y_k \in \mathbb{R}^r$, the state transition matrix $A \in \mathbb{R}^{n\times n}$ and the observation matrix $C \in \mathbb{R}^{r\times n}$, where r is the number of outputs and n is the system order, which is assumed to be known or estimated in the reference condition as in Bauer (2001). The process noise v_k is assumed to be stationary Gaussian white noise with zero mean and covariance matrix $Q : \mathbf{E}(v_k v_{k'}^T) \stackrel{\text{def}}{=} Q\,\delta(k-k')$, where $\mathbf{E}(\cdot)$ denotes the expectation operator, and w_k is the output noise. While the algorithms in this paper can be easily extended to a state–space model also containing input terms, we use the output-only model (1) for simplicity of notation and discuss the extension in Section 5.3.

The eigenstructure of system (1) shall be monitored for a change, which is the collection of eigenvalues and observed eigenvectors (λ, φ) with $A\phi_i = \lambda_i\phi_i$, $\varphi_i = C\phi_i$. The eigenstructure (λ, φ) is a canonical parameterization of system (1) that is invariant to the choice of its state–space basis and considered as the system parameter vector θ with

$$\theta \stackrel{\text{def}}{=} \begin{bmatrix} \Lambda \\ \text{vec}(\Phi) \end{bmatrix} \in \mathbb{C}^{(r+1)n}, \tag{2}$$

where $\Lambda = [\lambda_1 \cdots \lambda_n]^T$, $\Phi = [\varphi_1 \cdots \varphi_n]$ and vec denotes the vectorization operator. We assume that the system has no multiple eigenvalues.

2.2. Subspace-based residual function

For the detection of changes in the eigenstructure θ from a deterministic reference parameter θ_0 a residual function was proposed in Basseville et al. (2000) that is associated with a covariance-driven output-only subspace identification algorithm. Let $G = \mathbf{E}(x_{k+1}y_k^T)$ be the cross-covariance between the states and the outputs, let $R_i = \mathbf{E}(y_k y_{k-i}^T) = CA^{i-1}G$ $(i \geq 1)$ be the theoretic output covariances and $\mathcal{H}_{p+1,q}$ be the theoretic block Hankel matrix containing the R_i with the well-known factorization property

$$\mathcal{H}_{p+1,q} = \mathcal{O}_{p+1}\mathcal{C}_q \tag{3}$$

into observability $\mathcal{O}_{p+1} = [C^T\ (CA)^T \cdots (CA^p)^T]^T$ and controllability matrix $\mathcal{C}_q = [G\,AG \cdots A^{q-1}G]$. Parameters p and q are chosen such that $\text{rank}(\mathcal{O}_p) = \text{rank}(\mathcal{C}_q) = n$. For the construction of a residual function, the observability matrix $\mathcal{O}_{p+1}(\theta_0)$ is obtained from the reference model parameter θ_0 in the modal basis ($C = \Phi$, $A = \text{diag}(\Lambda)$). A matrix $S(\theta_0)$ is computed, whose columns are an orthonormal basis of the left null space of $\mathcal{O}_{p+1}(\theta_0)$, such that $S(\theta_0)^T\mathcal{O}_{p+1}(\theta_0) = 0$. Then, $S(\theta_0)$ also defines a basis of the left null space of $\mathcal{H}_{p+1,q}$ in the reference state because of factorization property (3), and the characteristic property of a system in the reference state corresponding to $\theta = \theta_0$ writes (Basseville et al., 2000)

$$S(\theta_0)^T\mathcal{H}_{p+1,q} = 0. \tag{4}$$

From the outputs $\mathcal{Y}_N \stackrel{\text{def}}{=} \{y_k : k = 1, \ldots, N\}$, a consistent estimate $\widehat{\mathcal{H}}_{p+1,q}$ is obtained from the estimated output covariances $\widehat{R}_i = 1/N\sum_{k=1}^N y_k y_{k-i}^T$.

To decide whether the measured data correspond to θ_0 or not, the residual vector

$$\zeta(\theta_0, \mathcal{Y}_N) = \sqrt{N}\,\text{vec}\left(S(\theta_0)^T\widehat{\mathcal{H}}_{p+1,q}\right) \tag{5}$$

is defined, which has the property

$$\mathbf{E}_\theta(\zeta(\theta_0, \mathcal{Y}_N)) = 0 \quad \text{iff} \quad \theta = \theta_0, \tag{6}$$

where $\mathbf{E}_\theta$ denotes the expectation operator when θ is the system parameter. This property can be tested with a hypothesis test derived in Basseville et al. (2000).

2.3. Residual properties and hypothesis test

In general, the distribution of $\zeta(\theta_0, \mathcal{Y}_N)$ is unknown when data is collected under parameter θ. However, an evaluation of the residual is possible based on the asymptotic local approach for change detection (Benveniste et al., 1987), assuming the close hypotheses

$$\begin{aligned} &\mathbf{H}_0 : \theta = \theta_0 \quad \text{(reference system)}, \\ &\mathbf{H}_1 : \theta = \theta_0 + \delta\theta/\sqrt{N} \quad \text{(faulty system)}, \end{aligned} \tag{7}$$

where vector $\delta\theta$ is unknown but fixed. With this statistical framework, very small changes in the system parameter θ can be detected if N is large enough. The statistical properties of the residual are analyzed for $N \to \infty$, and based on the asymptotic results