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Brief paper

Output-feedback finite-time stabilization of disturbed feedback linearizable nonlinear systems[☆]

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ABSTRACT

A novel methodology for designing multivariable High-Order Sliding-Mode (HOSM) controllers for disturbed feedback linearizable nonlinear systems is introduced. It provides for the finite-time stabilization of the origin of the state-space by using output feedback. Only the additional assumptions of algebraic strong observability and smooth enough matched disturbances are required. The control problem is solved in two consecutive steps: firstly, designing an observer based on the measured output and, secondly, designing of a full-state controller computed from a new virtual output with vector relative degree. The introduced notion of algebraic strong observability allows recovering the state of the system using the measured output and its derivatives. By estimating the required derivatives through the HOSM differentiator, a finite-time convergent observer is constructed.

1. Introduction

The design of robust controllers is an important topic in automatic control theory. Uncertainty is also manifested by only measuring a subset of the state variables of the system, namely, its "measured output". In such scenario, Sliding-Mode (SM) output-feedback based controllers have shown to be very successful (Oliveira, Peixoto, Costa, & Hsu, 2010; Oliveira, Peixoto, & Hsu, 2010, 2013; Peixoto, Oliveira, Hsu, Lizarralde, & Costa, 2011).

Moreover, modern systems frequently mix continuous and discrete event dynamics for which the stabilization problems are much more intricate. However, hybrid systems with strictly positive dwell-time can be effectively controlled if the controller accomplishes the control objective before the next switching or impulse time. Therefore, robust output-feedback controllers providing finite-time state stabilization become relevant.

High Order SMs (HOSMs) are useful in this context providing for the finite-time exact[2] output stabilization using only output feedback (Levant, 2003). In addition, in the presence of noise and sampling, HOSMs offer better accuracy than first-order SM. However, they were originally designed only for single-input single-output systems, see, e.g., Dinuzzo and Ferrara (2009) and Efimov, Zolghadri, and Raïssi (2011).

In Defoort, Floquet, Kokosy, and Perruquetti (2009), HOSM controllers are extended to Multi-Input Multi-Output (MIMO) nonlinear systems under the assumption of vector relative degree with respect to the measured output. This last assumption also requires that the system has the same number of inputs as outputs. Recently, in Angulo, Fridman, and Levant (2012), the authors introduced a methodology for the design of HOSM controllers for MIMO disturbed linear systems under necessary conditions: a known (affine) bound on the disturbance, controllability and strong observability (Hautus, 1983). The proposed methodology allows constructing an output-based HOSM controller guaranteeing the finite-time exact convergence of the whole state of the system.

This brief extends the methodology introduced in Angulo et al. (2012) in the context of linear systems to a class of MIMO nonlinear systems. As in the linear case, the measured output does not necessarily has vector relative degree. In this form, HOSMs can

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2 Exactness is more than robustness: the effect of (matched) disturbances is completely eliminated after a finite-time transient.

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be applied to a larger class of systems when compared to the strategy of Defoort et al. (2009). Our approach requires introducing a suitable concept of observability despite disturbances. The notion of "algebraic strong observability" corresponds to the possibility of reconstructing the state as a function only of the measured output and a finite number of its derivatives. By using the HOSM differentiator (Levant, 2003) to estimate the required derivatives, a finite-time convergent observer is obtained.

By combining the proposed observer with HOSMs controllers and dynamic feedback linearization, the problem of exact finite-time state stabilization of nonlinear systems is solved. The proofs of all Theorems are collected in the Appendix.

Main contributions. (1) An algorithm to construct an unknown input observer for nonlinear systems is presented based on the notion of algebraic strong observability and the use of HOSM differentiators; (2) by using the proposed observer, a novel HOSM-based output feedback control strategy for the finite-time state stabilization of (dynamic) feedback linearizable nonlinear systems with bounded matched disturbances is presented.

2. Problem statement

Consider

$$\dot{x} = f(x) + g(x)[u + w], \qquad y = h(x); \qquad x(0) = x_0, \tag{1}$$

where $x(t) \in \mathbb{R}^n$, $u(t) \in \mathbb{R}^m$, $w(t) \in \mathbb{R}^m$ and $y(t) \in \mathbb{R}^p$ are the state, control input, disturbance and measured output, respectively. The vector fields f, g and the function h are assumed to be smooth. The control objective is to design the control input $u(t)$, depending only on the output $y(t)$, to provide finite-time state stability at zero despite the presence of the disturbance $w(t)$.

In order to present the assumptions we have made to solve the control problem, let us first introduce the following concept:

Definition 1. System (1) is said to be algebraically strongly observable with respect to the output $y(t) \in \mathbb{R}^p$, if there exists a function F and integers k_i, $i = 1, \ldots, p$, such that

$$x = F(y_1, \ldots, y_1^{(k_1)}, \ldots, y_p, \ldots, y_p^{(k_p)}). \tag{2}$$

Strong observability was introduced by Hautus (1983) for linear systems to solve the Unknown Input Observer (UIO) design problem. It allows recovering the state of the system through the knowledge of the output and its derivatives only, irrespective of the input. The notion of algebraic strong observability is a natural extension to the case of nonlinear systems. In Section 3, this property will be shown to be instrumental for the design of a UIO for nonlinear systems.

The following assumptions about the system will be made throughout this paper:

(A1) system (1) is algebraically strongly observable with respect to the measured output y;

(A2) for a given degree of smoothness $S \in \mathbb{Z}_+$, there exists a constant W^+ such that $\|w^{(k)}(t)\| \le W^+, \forall t \ge 0$ and all $k = 0, \ldots, S$;

(A3) for all initial conditions $x_0 \in \mathbb{R}^n$ and disturbances $w(\cdot)$ satisfying A2, the solution[3] to system (1) with $u(\cdot) \equiv 0$ exists for all $t \ge 0$ and remains upper-bounded by a possibly unknown number;

(A4) there exists a function $q : \mathbb{R}^n \to \mathbb{R}^m$ such that $z = q(x)$ is a flat output for system (1).

A flat system is feedback linearizable and, moreover, its state and input can be written as a function of the flat output and its derivatives (Sira Ramírez & Agrawal, 2004). Without loss of generality, we assume that $q(0) = 0$. Note that, in general, the flat output $z(t)$ may not coincide with the measured output $y(t)$. The degree S of required smoothness for the disturbance depends on the "order" of the dynamic compensator used to linearize the system, as shown in Section 4. In the particular case when the system can be linearized by a static compensator, the disturbance needs to be only uniformly bounded.

The approach we follow to solve the problem involves two steps. First, based on A1, we present an algorithm to compute the function F appearing in Eq. (2). This allows constructing a UIO for the system once the required derivatives are estimated. With Assumptions A2 and A3, it becomes possible to estimate such derivatives using the HOSM differentiator (Levant, 2003). In the second step, by using A4, it is shown that the finite-time state stabilization problem is equivalent to the finite-time stabilization of the flat output. This way, by using the estimated state of the previous step to evaluate the flat output, a multivariable HOSM controller that semi-globally stabilizes the flat output to zero in finite-time is presented.

3. Construction of the Unknown Input Observer

Once the function F in (2) is known, the state can be determined when the derivatives of the output are estimated. By means of the HOSM differentiator, these derivatives can be estimated exactly and in finite time. In addition, the HOSM differentiator provides an estimation that is robust to measurement noise and has the best order of precision (Levant, 2003).

The following Subsection presents an algorithm to compute the function F for a class of nonlinear systems. Section 3.2, analyzes the use of the HOSM differentiator to estimate the required derivatives.

3.1. An algorithm to construct an Unknown Input Observer

Let us consider the following additional assumption:

(A5) The Lie derivatives $L_g L_f^k h(x)$ are constant for $k = 1, \ldots, n-1$.

This assumption means that in the time derivatives of the measured output, the matrix multiplying the input is constant. This allows writing a constant orthogonal to such matrix. Assumption A5 is only introduced to simplify the presentation of the algorithm: in the general case when A5 is not satisfied, the orthogonal can be computed in the same way but now depending on the state.

Let $b^\perp$ denote a left annihilator for matrix b (i.e. $b^\perp b = 0$). The presented algorithm computes the function F introduced in Eq. (2), enabling the construction of a UIO for system (1). Our algorithm is the "dual" of the standard Structure Algorithm (see Conte, Moog, & Perdon, 2007, p. 76), since it looks for directions orthogonal to the input. Nevertheless, it is closely related to the algorithm presented in Molinari (1976) for linear systems.

Step 0. Set $\hat{y}_0 := y$ and $M_0(x) := h(x)$.

Step 1. Compute $\dot{M}_0(x) = \dot{\hat{y}}_0 = L_f M_0(x) + L_g M_0(x) w := a_1(x) + b_1(u + w)$, introduce $\dot{\hat{y}}_1 := b_1^\perp \dot{M}_0(x) = \hat{a}_1(x)$ and set

$$M_1(x) := \begin{bmatrix} \hat{y}_0 \\ \dot{\hat{y}}_1 \end{bmatrix} = \begin{bmatrix} h(x) \\ \hat{a}_1(x) \end{bmatrix}.$$

Step $k+1$. Compute

$$\dot{M}_k(x) = \begin{bmatrix} \dot{\hat{y}}_0 & \cdots & \hat{y}_k^{(k+1)} \end{bmatrix}^T := a_{k+1}(x) + b_{k+1}(u + w),$$

introduce $\hat{y}_{k+1}^{(k+1)} := b_{k+1}^\perp \dot{M}_k(x) = \hat{a}_{k+1}(x)$, and set

$$M_{k+1}(x) := \begin{bmatrix} \hat{y}_0 \\ \vdots \\ \hat{y}_{k+1}^{(k+1)} \end{bmatrix} = \begin{bmatrix} h(x) \\ \vdots \\ \hat{a}_{k+1}(x) \end{bmatrix}.$$

By applying the algorithm, one is able to compute a sequence of equations of the form $[\hat{y}_0, \dot{\hat{y}}_1, \ldots, \hat{y}_k^{(k)}]^T = M_k(x)$ for $k \ge 1$. The algorithm converges in the following sense.

[3] Solutions to differential equations (and their associated inclusions) are understood in Filippov's sense (Filippov, 1960).